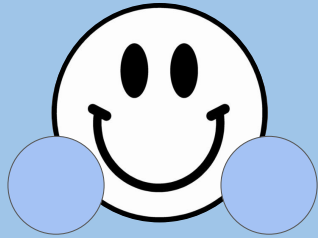
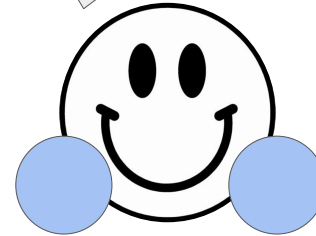


THE ADVENTURES OF SCOTT THE WATER MOLECULE: BOUNDARY CONDITIONS

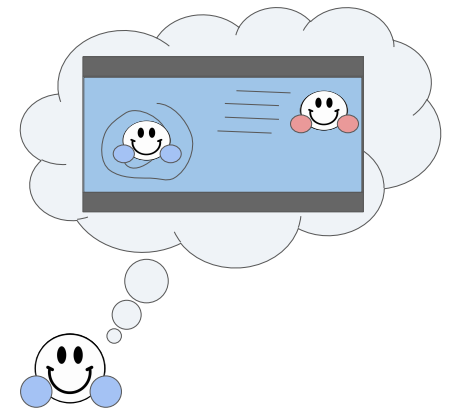


Written by Lina Abu-Absi
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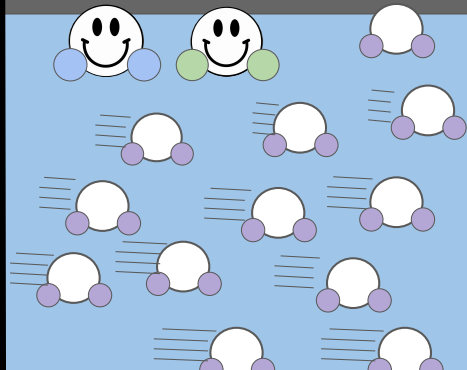
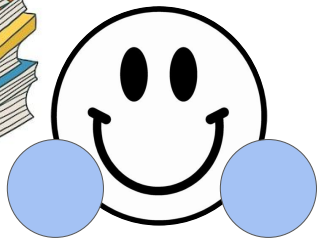
Hi, I'm Scott the Water Molecule! I'm on a mission to find my friend, Jerry.



Not too long ago, Scott and Jerry were traveling along a pipe when suddenly, they got separated! Scott got caught in an eddy and Jerry was nowhere to be seen...

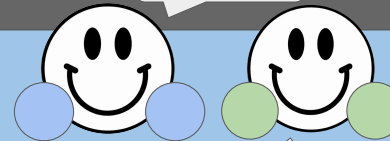


Now, I'm trying to do everything I can to learn more about transport phenomena!



One day, Scott was hanging out next to a plate with his friend Jenny when the bottom plate started moving. Scott noticed that all the other water molecules were moving with the bottom plate.

Jenny, where's everyone going?

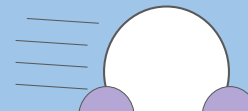


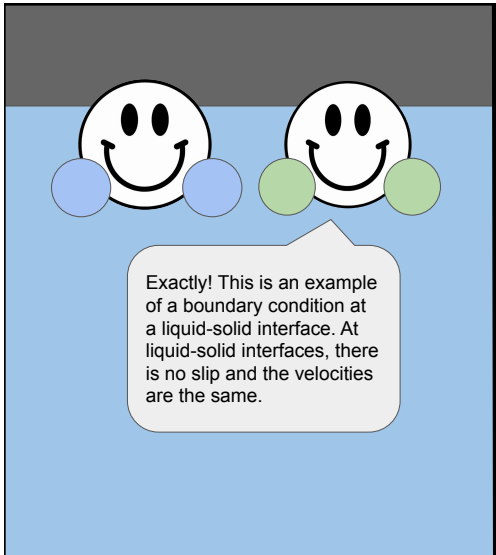
Haven't you heard of the no-slip condition?

No, of course not! What's that?

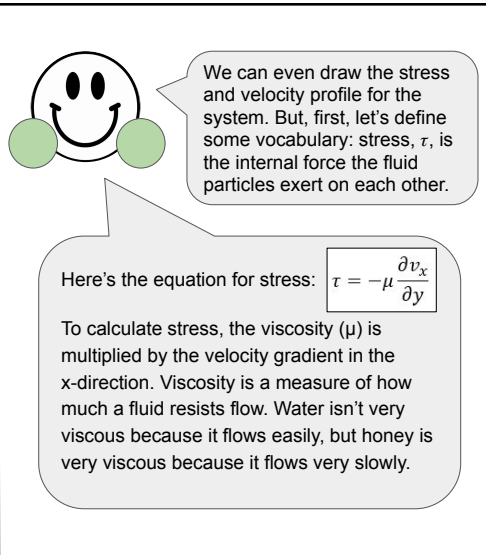
The no-slip condition is when fluids "stick" to boundaries. So, because the wall we're next to isn't moving, we're not moving either.

Oh, okay. So, if the wall was moving, we'd be moving with it?





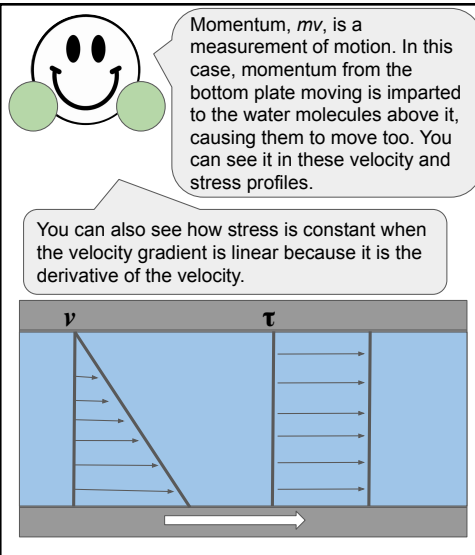
Exactly! This is an example of a boundary condition at a liquid-solid interface. At liquid-solid interfaces, there is no slip and the velocities are the same.



We can even draw the stress and velocity profile for the system. But, first, let's define some vocabulary: stress, τ , is the internal force the fluid particles exert on each other.

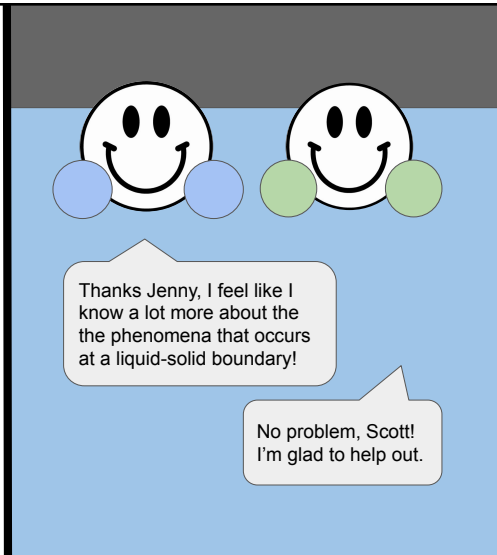
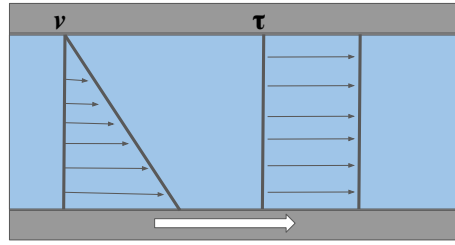
Here's the equation for stress:
$$\tau = -\mu \frac{\partial v_x}{\partial y}$$

To calculate stress, the viscosity (μ) is multiplied by the velocity gradient in the x-direction. Viscosity is a measure of how much a fluid resists flow. Water isn't very viscous because it flows easily, but honey is very viscous because it flows very slowly.



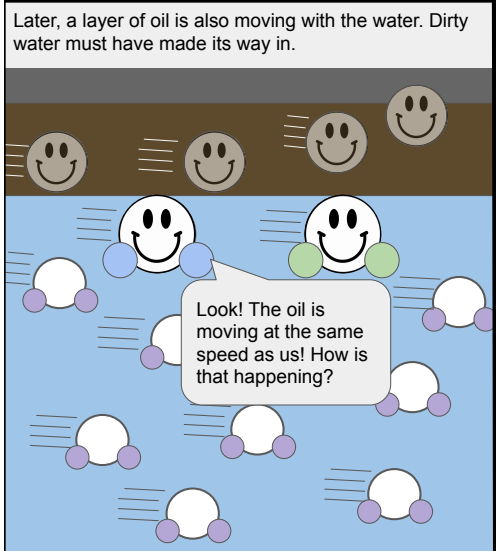
Momentum, mv , is a measurement of motion. In this case, momentum from the bottom plate moving is imparted to the water molecules above it, causing them to move too. You can see it in these velocity and stress profiles.

You can also see how stress is constant when the velocity gradient is linear because it is the derivative of the velocity.



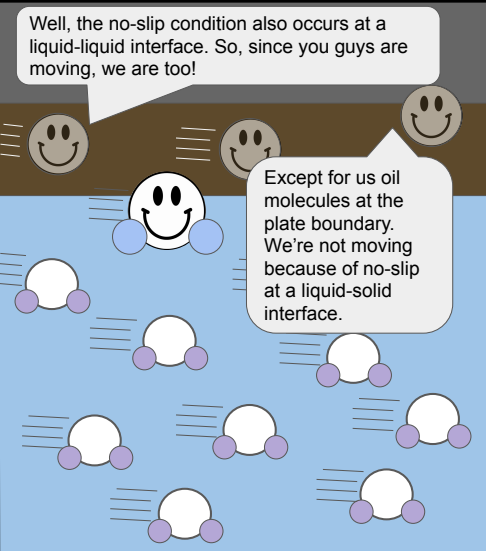
Thanks Jenny, I feel like I know a lot more about the phenomena that occurs at a liquid-solid boundary!

No problem, Scott! I'm glad to help out.



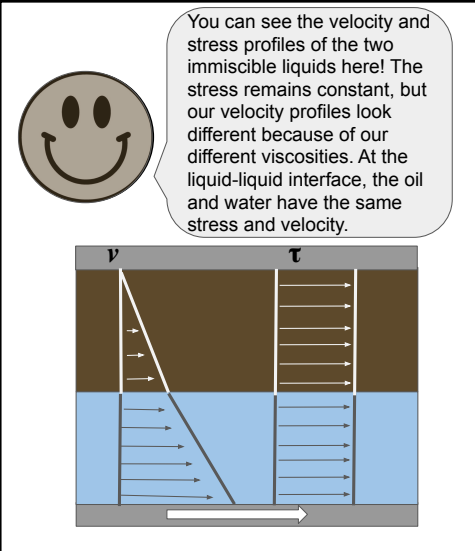
Later, a layer of oil is also moving with the water. Dirty water must have made its way in.

Look! The oil is moving at the same speed as us! How is that happening?

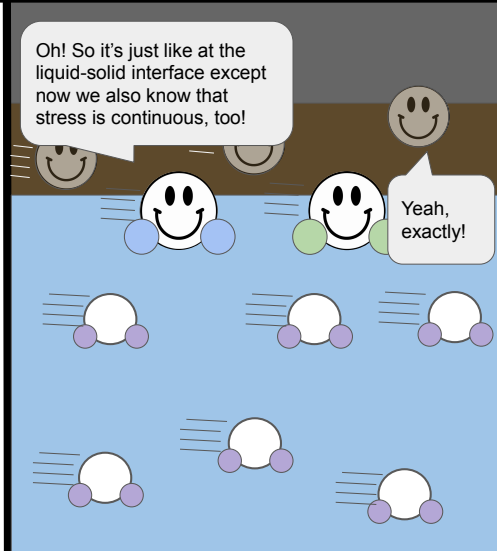
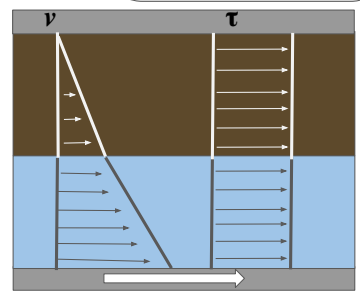


Well, the no-slip condition also occurs at a liquid-liquid interface. So, since you guys are moving, we are too!

Except for us oil molecules at the plate boundary. We're not moving because of no-slip at a liquid-solid interface.



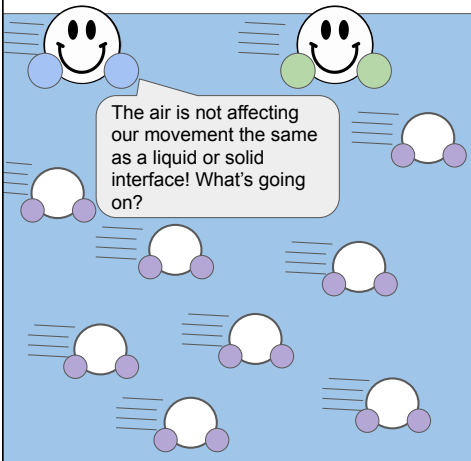
You can see the velocity and stress profiles of the two immiscible liquids here! The stress remains constant, but our velocity profiles look different because of our different viscosities. At the liquid-liquid interface, the oil and water have the same stress and velocity.



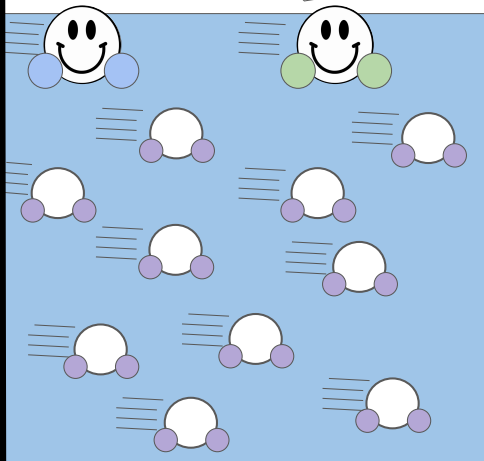
Oh! So it's just like at the liquid-solid interface except now we also know that stress is continuous, too!

Yeah, exactly!

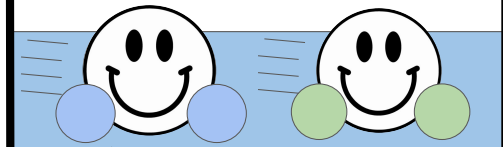
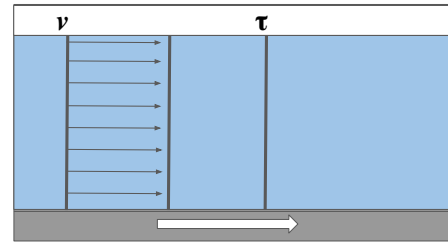
A few days later, the pipe emptied out into a pond and Scott and Jenny were at the surface.



That's because there's no resistance for momentum at a gas-liquid boundary! As a result, there isn't a velocity gradient or any stress.

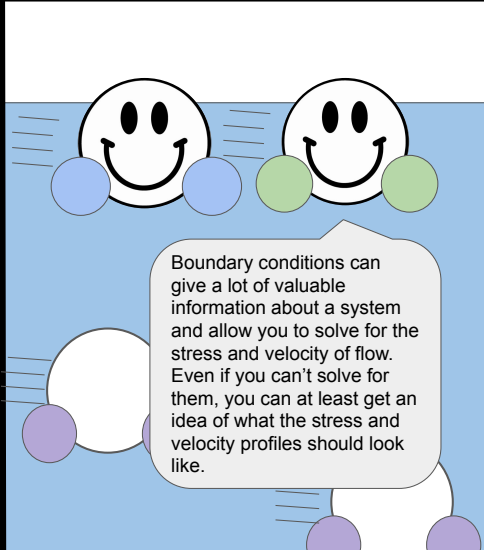
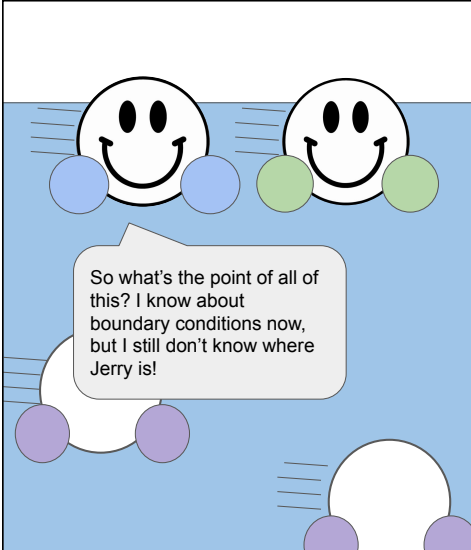


Here you can see the velocity and stress profiles for a situation with a liquid-gas interface and a solid-liquid interface, where the bottom plate is moving. The velocity is constant, so the stress is 0.



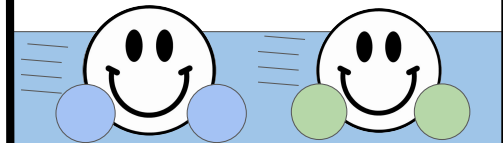
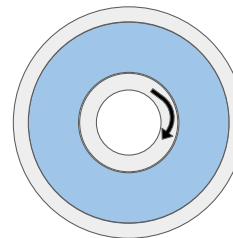
That makes sense. So at a liquid-gas interface, the stress and velocity gradient are always zero?

Yeah, you're getting the hang of it now!



Let's say we have an incompressible fluid flowing in the annulus of two cylinders. We can start to describe what's going on using what we know about boundary conditions!

The inner cylinder is moving with an angular velocity and the outer cylinder is stationary.



Since we know that there's no slip at a fluid-solid interface, that means the velocity of the fluid at the outer wall isn't moving, but at the inner wall, the fluid has the same velocity as the wall.

Exactly. And we could combine this information with other assumptions to solve for the velocity and stress of the system. But, that's for another day...

