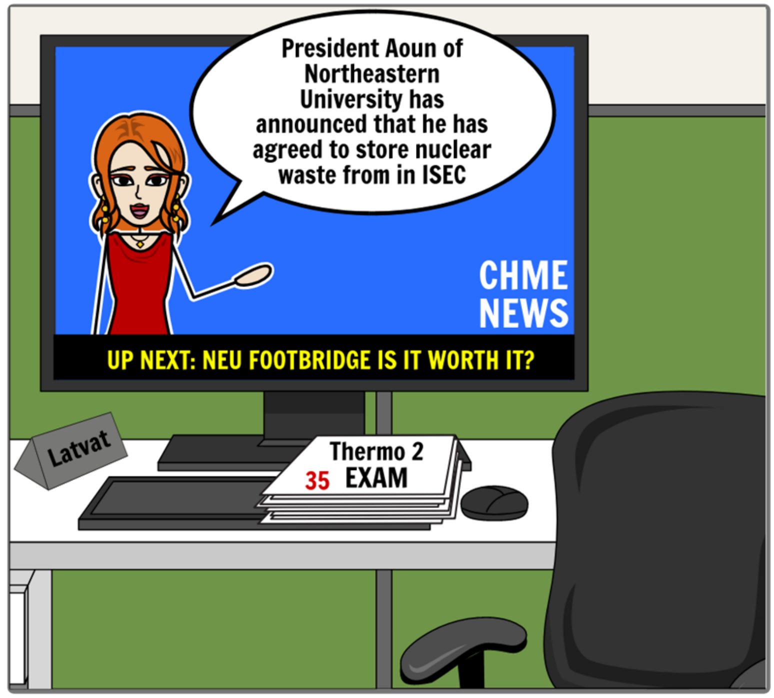
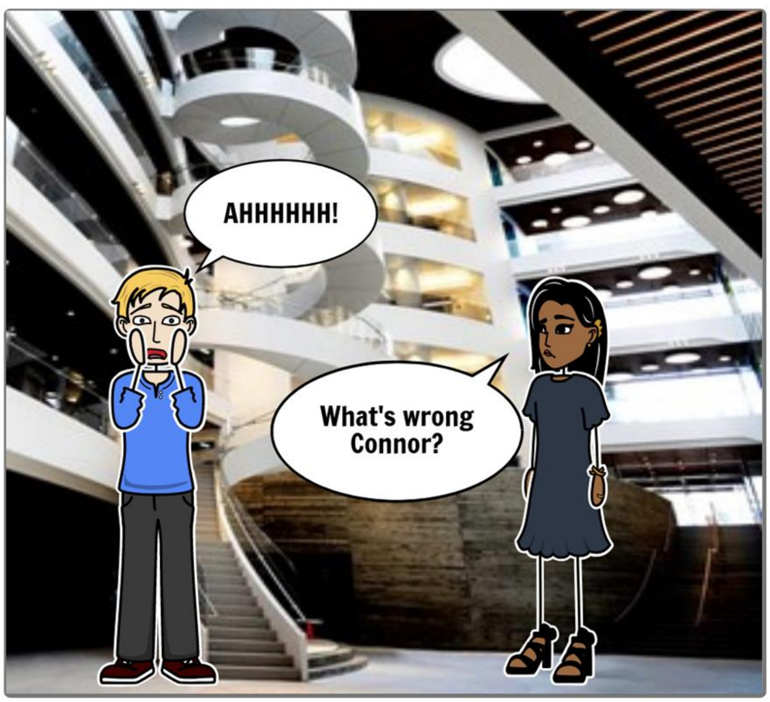
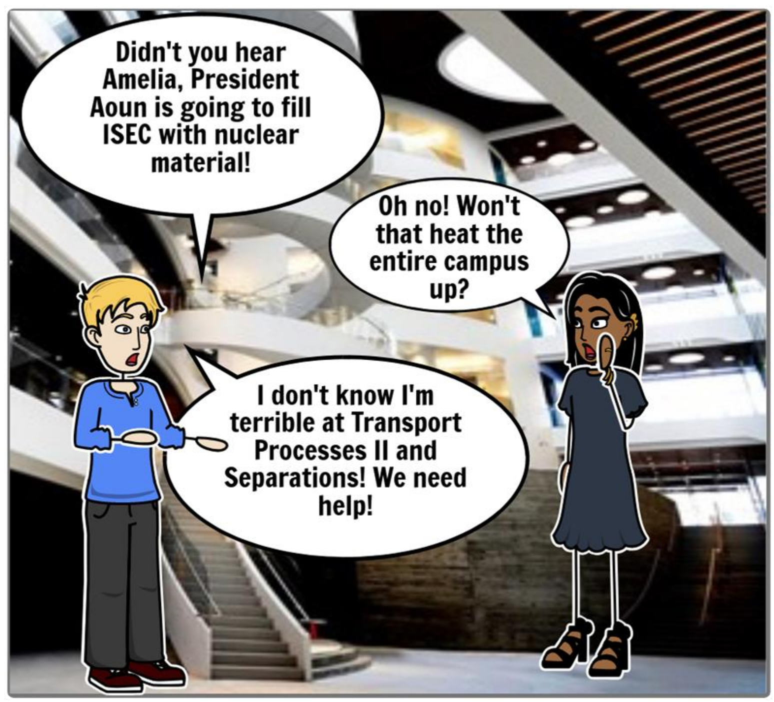
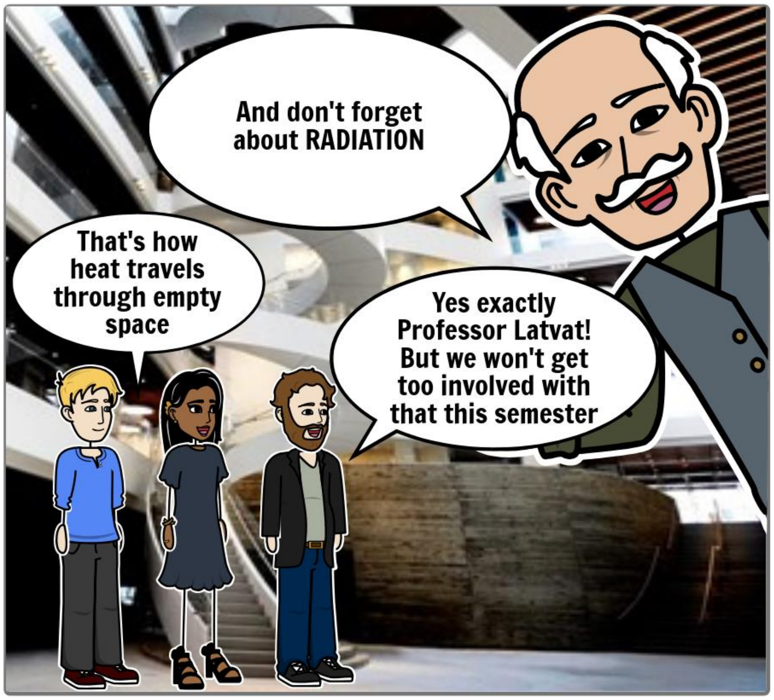

ADVENTURES IN ISEC
INTRODUCTION TO CONDUCTION AND CONVECTION
FEATURING PROFESSOR SANDHERR AND PROFESSOR LATVAT
BY: LUKAS MORGAN AND ANDREW MCGLYNN

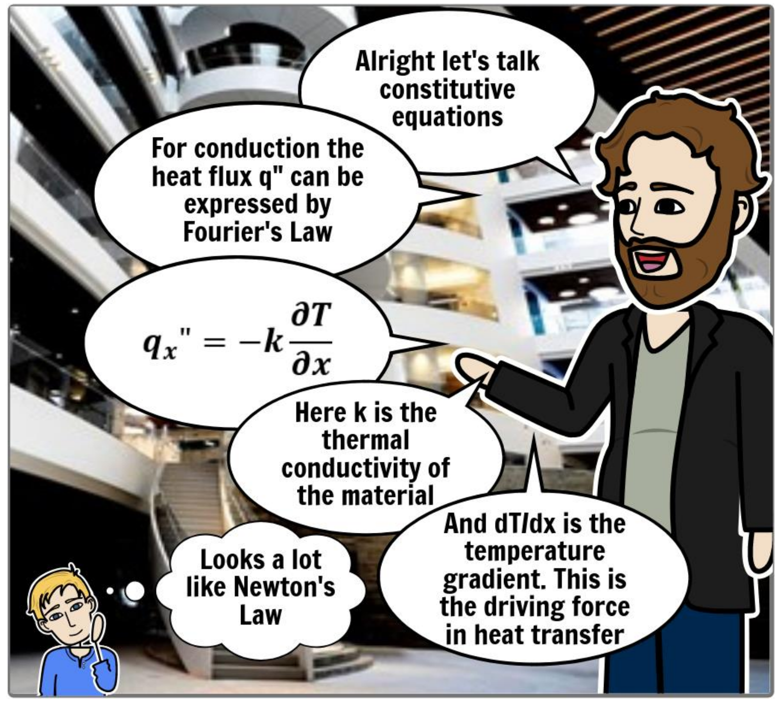

President Aoun of Northeastern University has announced that he has agreed to store nuclear waste from in ISEC
CHME NEWS
UP NEXT: NEU FOOTBRIDGE IS IT WORTH IT?
Latvat
Thermo 2
35 EXAM

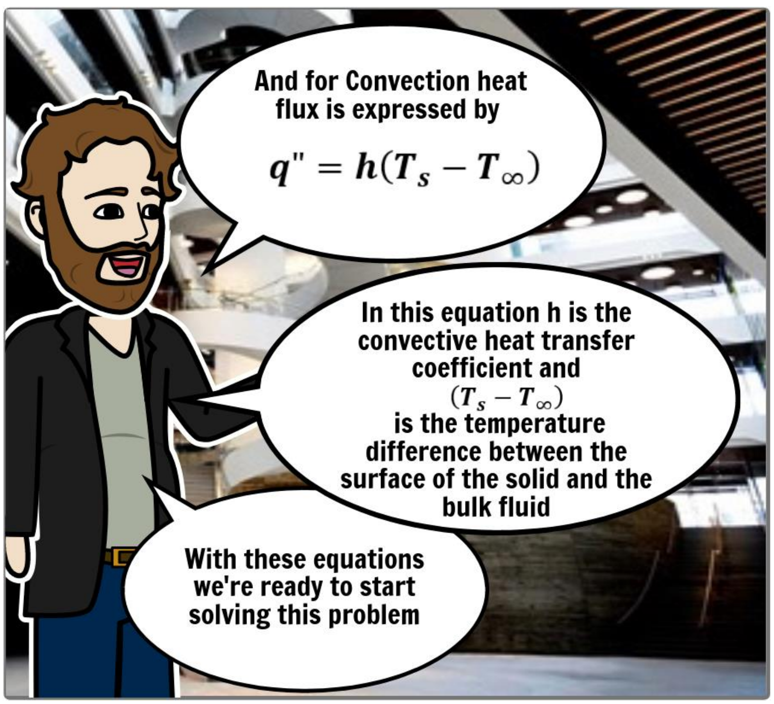

AHHHHHH!
What's wrong Connor?

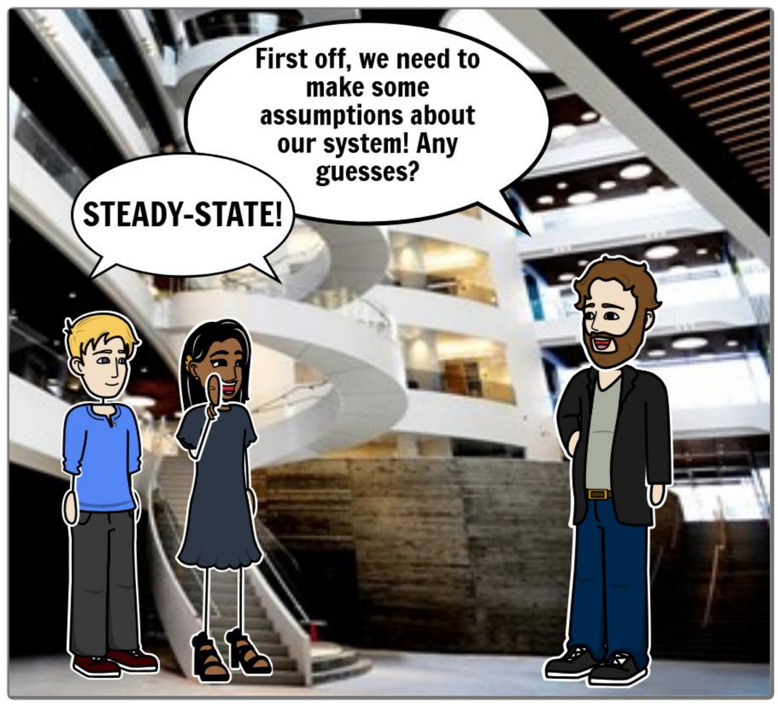

Didn't you hear Amelia, President Aoun is going to fill ISEC with nuclear material!
Oh no! Won't that heat the entire campus up?
I don't know I'm terrible at Transport Processes II and Separations! We need help!

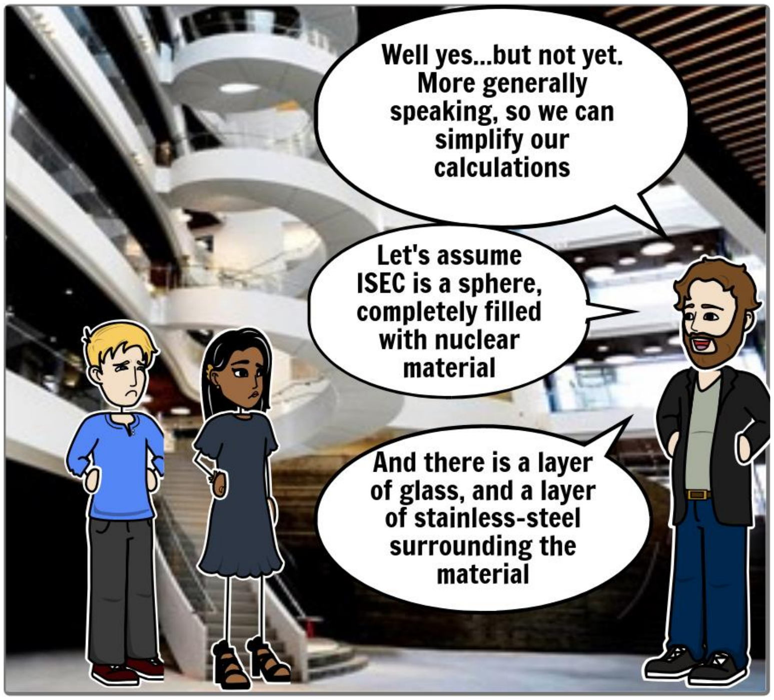

Hooray! It's Professor Sandherr!
Alright who's ready to learn about Heat Transfer!
Well I'm 3 years into this degree and it's way to late to switch so what choice do I really have
Okay so for starters can either of you tell me what the 3 modes of heat transfer are?
Hmmmmm
Oh I know, when two surfaces are in contact they exchange heat!
Excellent that is called CONDUCTION
What about heat transfer between a fluid and a solid surface?
Yes! You're referring to CONVECTION


That's how heat travels through empty space
And don't forget about RADIATION
Yes exactly Professor Latvat! But we won't get too involved with that this semester


Alright let's talk constitutive equations
For conduction the heat flux q" can be expressed by Fourier's Law
$q_x" = -k\frac{\partial T}{\partial x}$
Here k is the thermal conductivity of the material
Looks a lot like Newton's Law
And dT/dx is the temperature gradient. This is the driving force in heat transfer

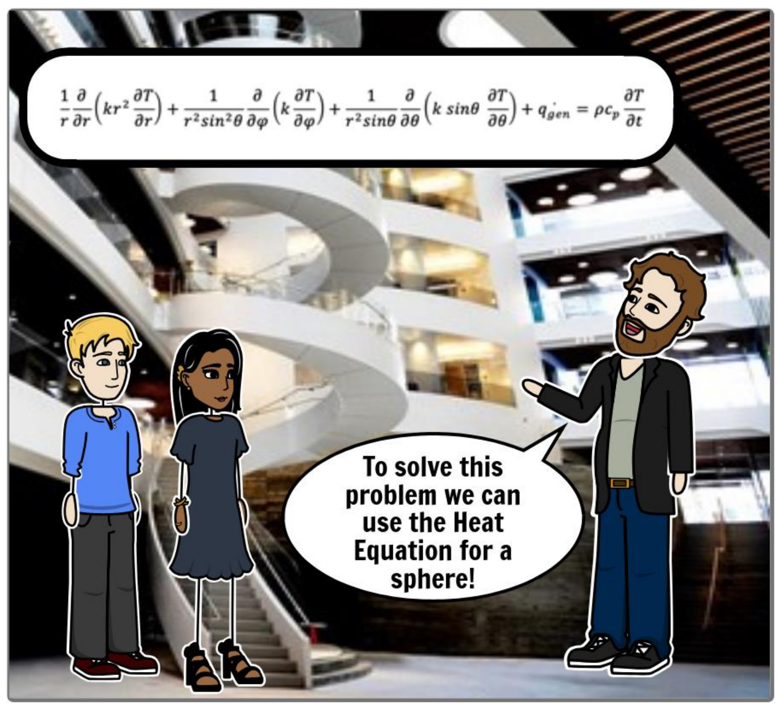

And for Convection heat flux is expressed by
$q" = h(T_s - T_\infty)$
In this equation h is the convective heat transfer coefficient and $(T_s - T_\infty)$ is the temperature difference between the surface of the solid and the bulk fluid
With these equations we're ready to start solving this problem

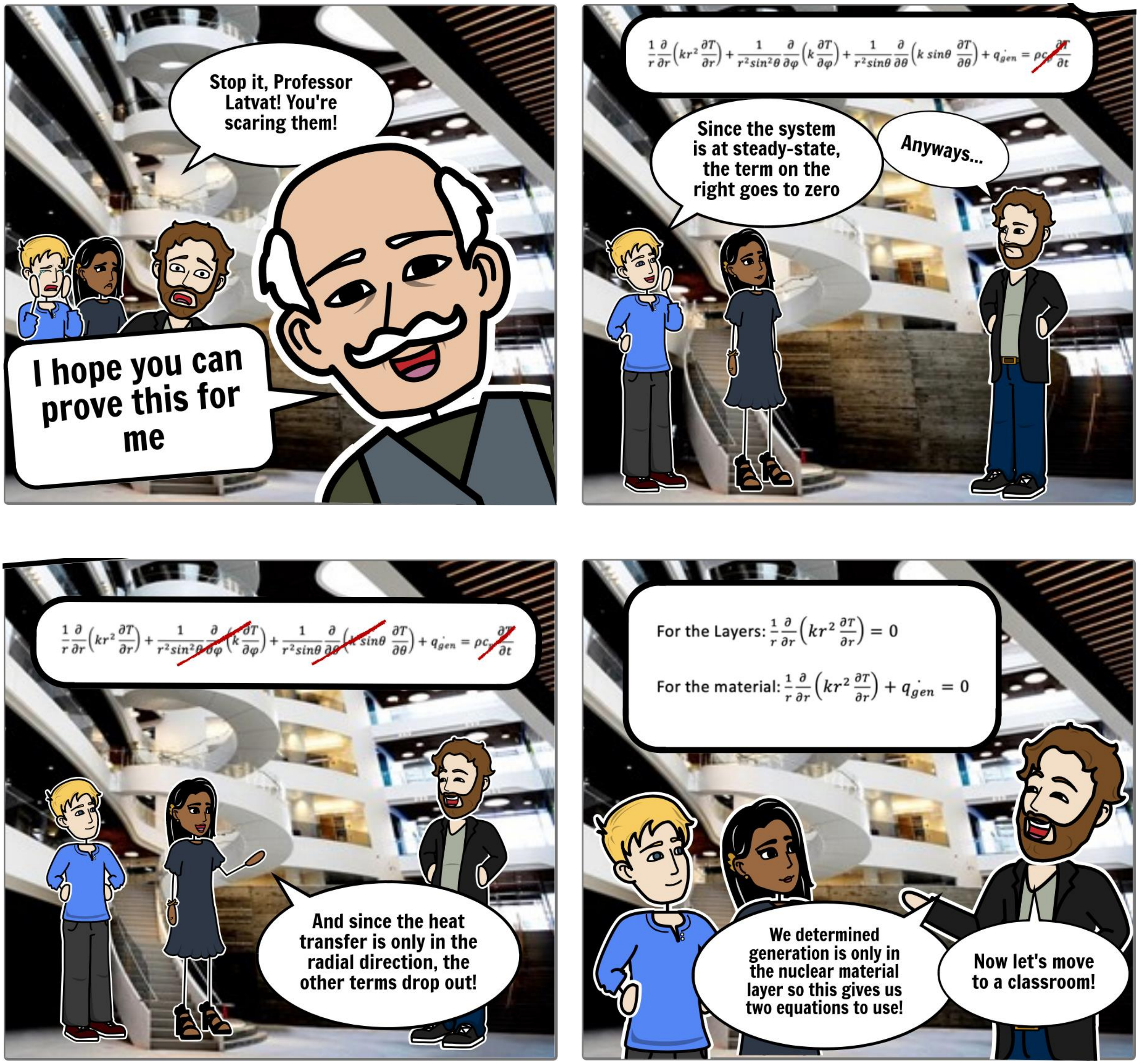

First off, we need to make some assumptions about our system! Any guesses?
STEADY-STATE!


Well yes...but not yet. More generally speaking, so we can simplify our calculations
Let's assume ISEC is a sphere, completely filled with nuclear material
And there is a layer of glass, and a layer of stainless-steel surrounding the material

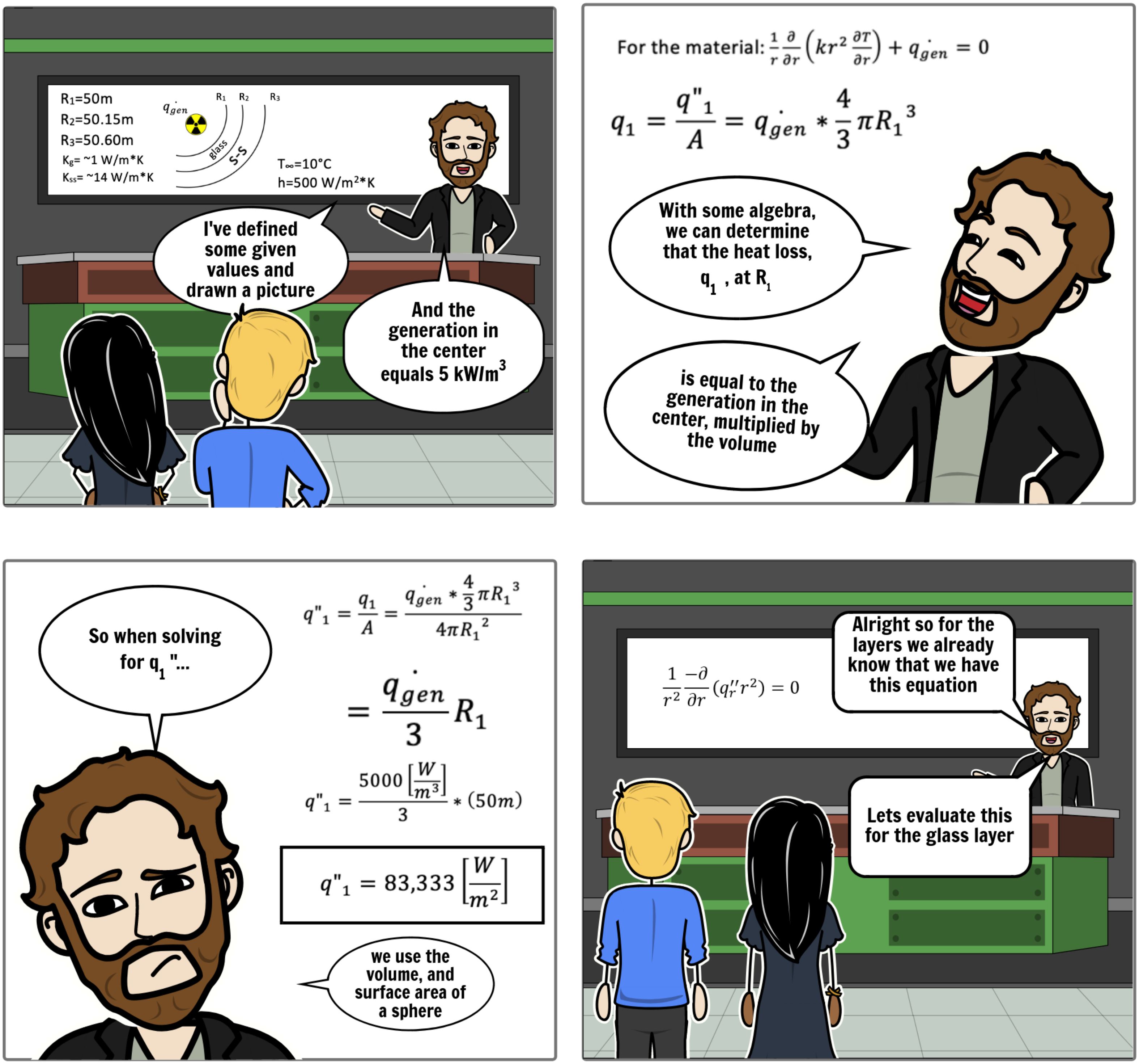

Also there's only generation of heat in the material at the center
And none in the glass and stainless-steel layers


With these assumptions, we can simplify our general equation and start solving!

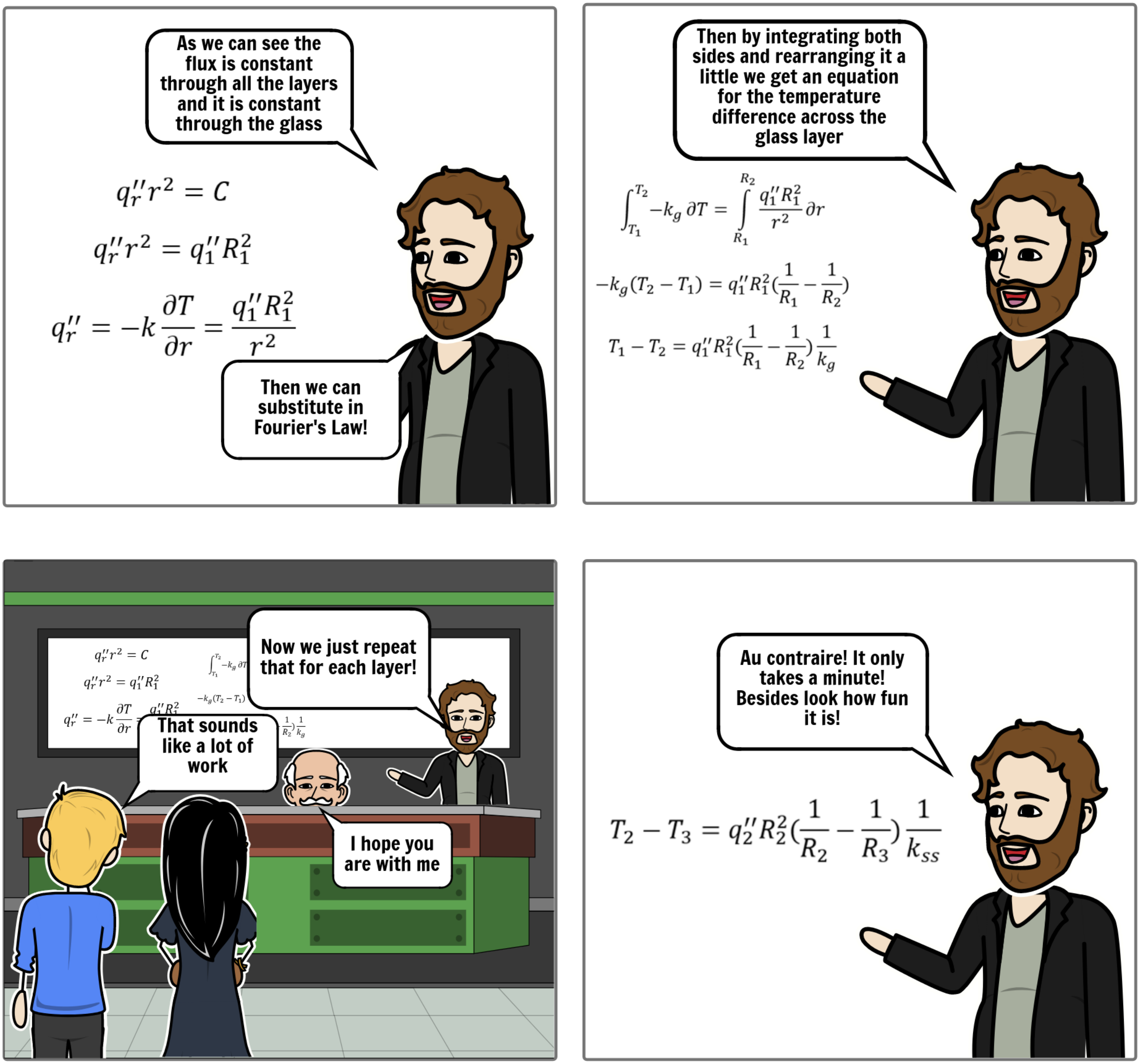

$\frac{1}{r}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial\varphi}\left(k\frac{\partial T}{\partial\varphi}\right)+\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(k\,sin\theta\,\frac{\partial T}{\partial\theta}\right)+\dot{q}_{gen}=\rho c_p\frac{\partial T}{\partial t}$
To solve this problem we can use the Heat Equation for a sphere!

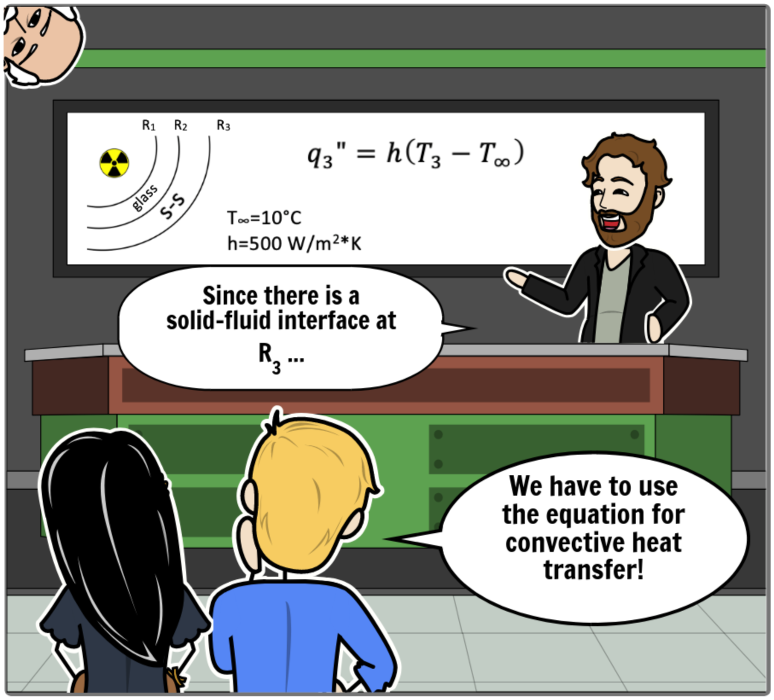


Stop it, Professor Latvat! You're scaring them!
I hope you can prove this for me
$\frac{1}{r}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial\varphi}\left(k\frac{\partial T}{\partial\varphi}\right)+\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(k\, sin\theta\,\frac{\partial T}{\partial\theta}\right)+\dot{q_{gen}}=\cancel{\rho c_p\frac{\partial T}{\partial t}}$
Since the system is at steady-state, the term on the right goes to zero
Anyways...
$\frac{1}{r}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\cancel{\frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial\varphi}\left(k\frac{\partial T}{\partial\varphi}\right)}+\cancel{\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(k\, sin\theta\,\frac{\partial T}{\partial\theta}\right)}+\dot{q_{gen}}=\cancel{\rho c_p\frac{\partial T}{\partial t}}$
And since the heat transfer is only in the radial direction, the other terms drop out!
For the Layers: $\frac{1}{r}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)=0$
For the material: $\frac{1}{r}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\dot{q_{gen}}=0$
We determined generation is only in the nuclear material layer so this gives us two equations to use!
Now let's move to a classroom!

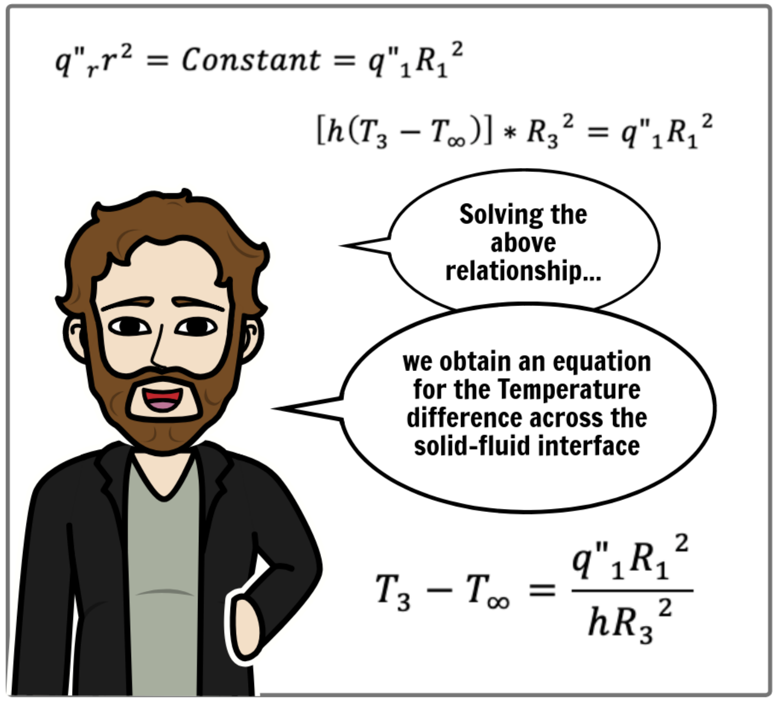


R_1=50m
R_2=50.15m
R_3=50.60m
K_g= ~1 W/m*K
K_{ss}= ~14 W/m*K
$\dot{q}_{gen}$
R_1 R_2 R_3
glass
S-S
T_∞=10°C
h=500 W/m²*K
I've defined some given values and drawn a picture
And the generation in the center equals 5 kW/m³
For the material: $\frac{1}{r}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \dot{q}_{gen} = 0$
$q_1 = \frac{q"_1}{A} = \dot{q}_{gen} * \frac{4}{3}\pi {R_1}^3$
With some algebra, we can determine that the heat loss, q_1, at R_1
is equal to the generation in the center, multiplied by the volume
So when solving for q_1"...
$q"_1 = \frac{q_1}{A} = \frac{\dot{q}_{gen} * \frac{4}{3}\pi {R_1}^3}{4\pi {R_1}^2}$
$= \frac{\dot{q}_{gen}}{3} R_1$
$q"_1 = \frac{5000\left[\frac{W}{m^3}\right]}{3} * (50m)$
$q"_1 = 83{,}333\left[\frac{W}{m^2}\right]$
we use the volume, and surface area of a sphere
$\frac{1}{r^2}\frac{-\partial}{\partial r}(q_r'' r^2) = 0$
Alright so for the layers we already know that we have this equation
Lets evaluate this for the glass layer

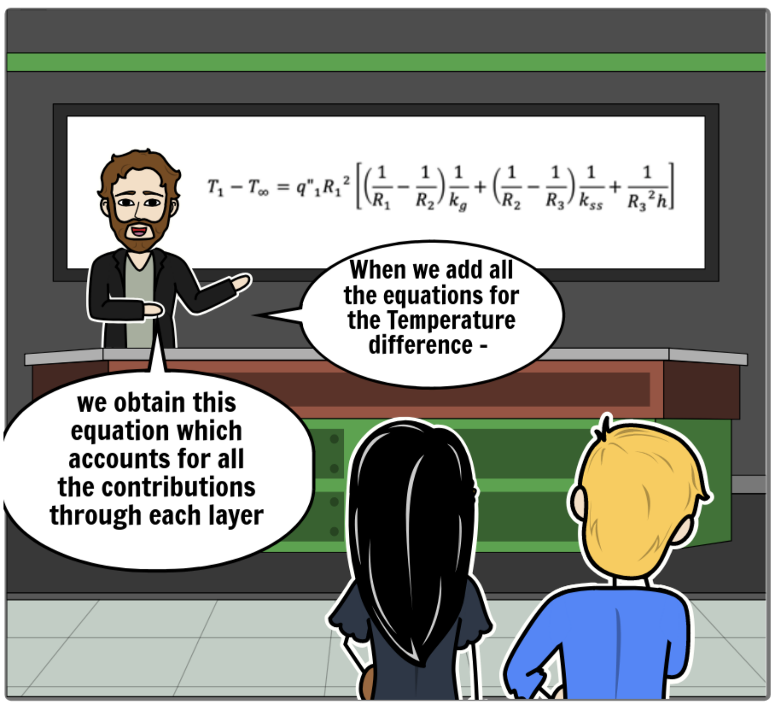

As we can see the flux is constant through all the layers and it is constant through the glass
$q_r''r^2 = C$
$q_r''r^2 = q_1''R_1^2$
$q_r'' = -k\frac{\partial T}{\partial r} = \frac{q_1''R_1^2}{r^2}$
Then we can substitute in Fourier's Law!
Then by integrating both sides and rearranging it a little we get an equation for the temperature difference across the glass layer
$\int_{T_1}^{T_2} -k_g\,\partial T = \int_{R_1}^{R_2} \frac{q_1''R_1^2}{r^2}\partial r$
$-k_g(T_2 - T_1) = q_1''R_1^2(\frac{1}{R_1} - \frac{1}{R_2})$
$T_1 - T_2 = q_1''R_1^2(\frac{1}{R_1} - \frac{1}{R_2})\frac{1}{k_g}$
Now we just repeat that for each layer!
That sounds like a lot of work
I hope you are with me
Au contraire! It only takes a minute! Besides look how fun it is!
$T_2 - T_3 = q_2''R_2^2(\frac{1}{R_2} - \frac{1}{R_3})\frac{1}{k_{ss}}$



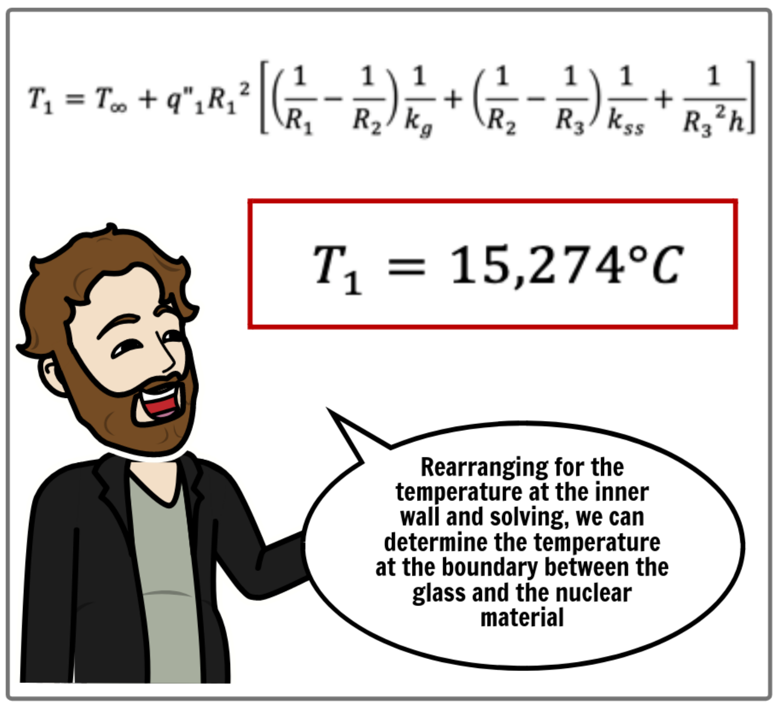
R_1 R_2 R_3
glass
S-S
$q_3" = h(T_3 - T_\infty)$
T_∞=10°C
h=500 W/m²*K
Since there is a solid-fluid interface at R_3 ...
We have to use the equation for convective heat transfer!



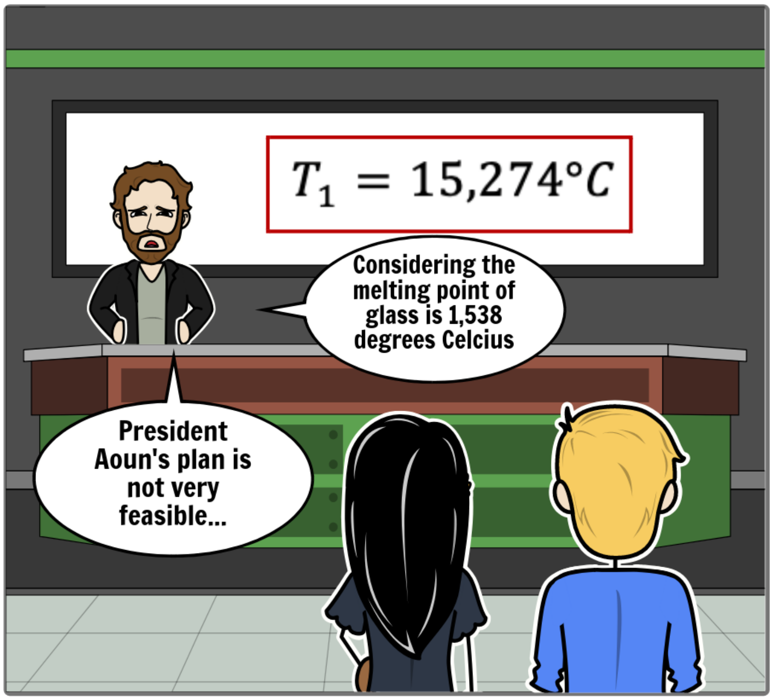
$q"_r r^2 = Constant = q"_1 {R_1}^2$
$[h(T_3 - T_\infty)] * {R_3}^2 = q"_1 {R_1}^2$
Solving the above relationship...
we obtain an equation for the Temperature difference across the solid-fluid interface
$T_3 - T_\infty = \frac{q"_1 {R_1}^2}{h {R_3}^2}$

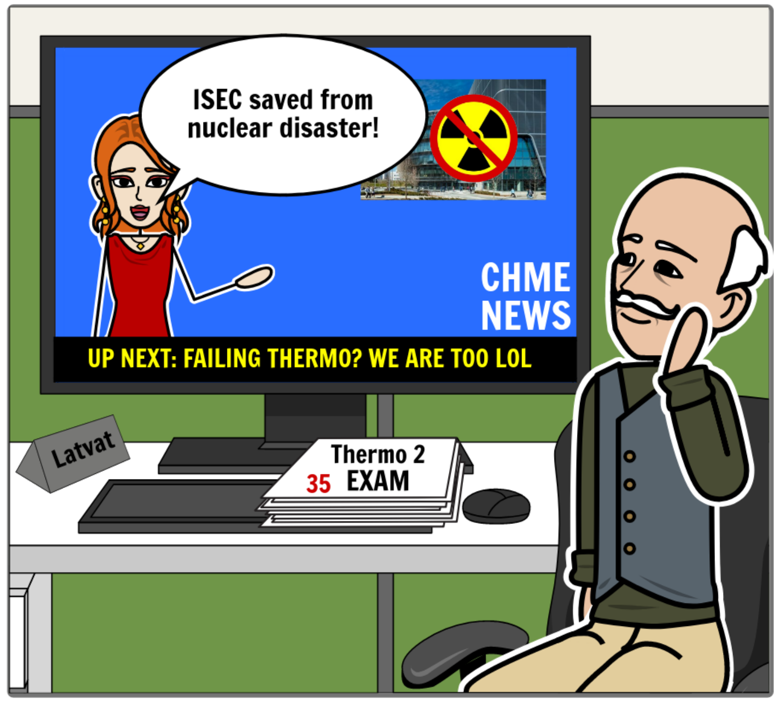


$T_1 - T_\infty = q"_1 {R_1}^2 \left[\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{1}{k_g} + \left(\frac{1}{R_2} - \frac{1}{R_3}\right)\frac{1}{k_{ss}} + \frac{1}{{R_3}^2 h}\right]$
When we add all the equations for the Temperature difference -
we obtain this equation which accounts for all the contributions through each layer



$T_1 = T_\infty + q"_1 {R_1}^2 \left[\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{1}{k_g} + \left(\frac{1}{R_2} - \frac{1}{R_3}\right)\frac{1}{k_{ss}} + \frac{1}{{R_3}^2 h}\right]$
$T_1 = 15{,}274°C$
Rearranging for the temperature at the inner wall and solving, we can determine the temperature at the boundary between the glass and the nuclear material


$T_1 = 15{,}274°C$
Considering the melting point of glass is 1,538 degrees Celcius
President Aoun's plan is not very feasible...


ISEC saved from nuclear disaster!
CHME NEWS
UP NEXT: FAILING THERMO? WE ARE TOO LOL
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