Composite Walls: Bringing the Layers Together

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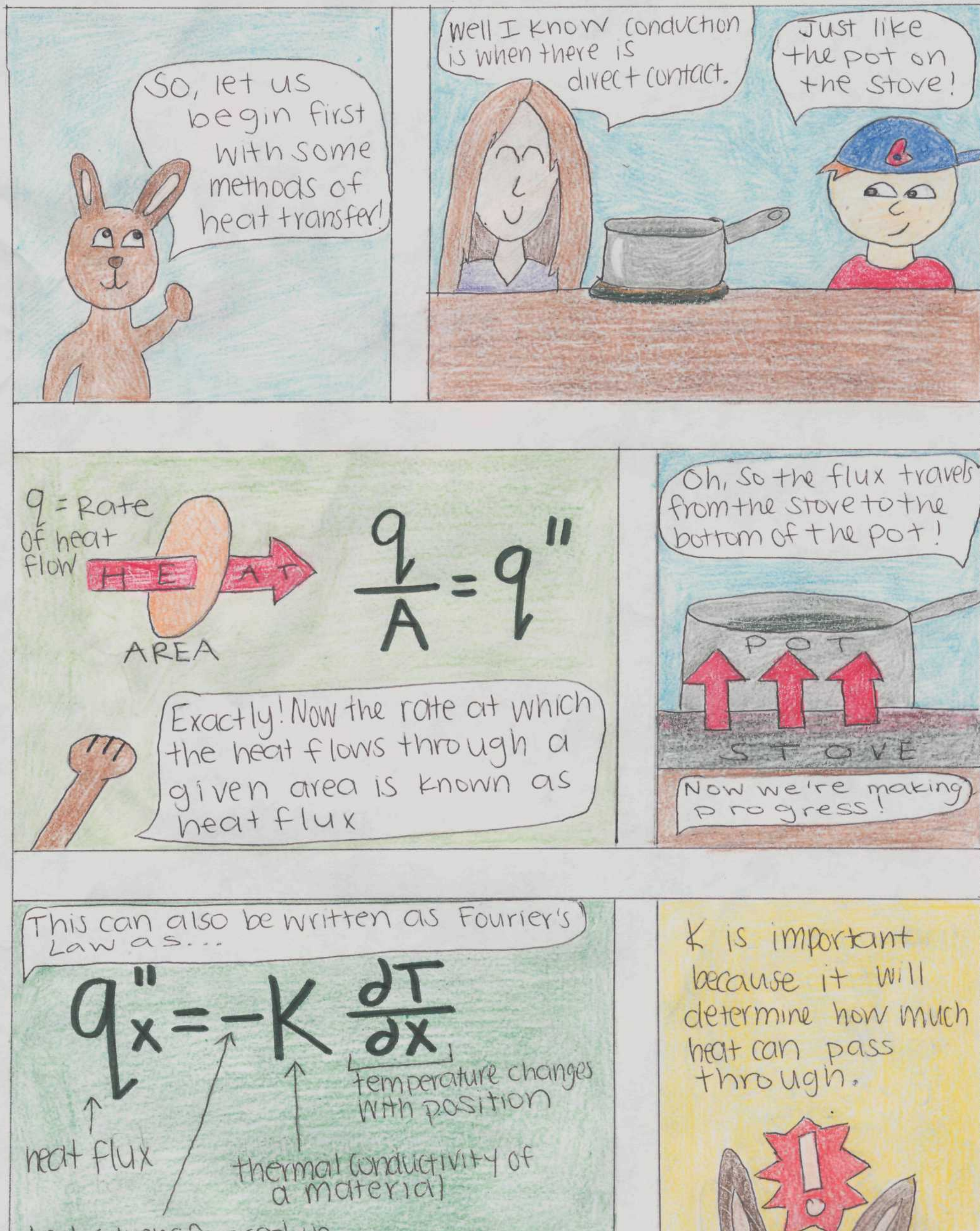
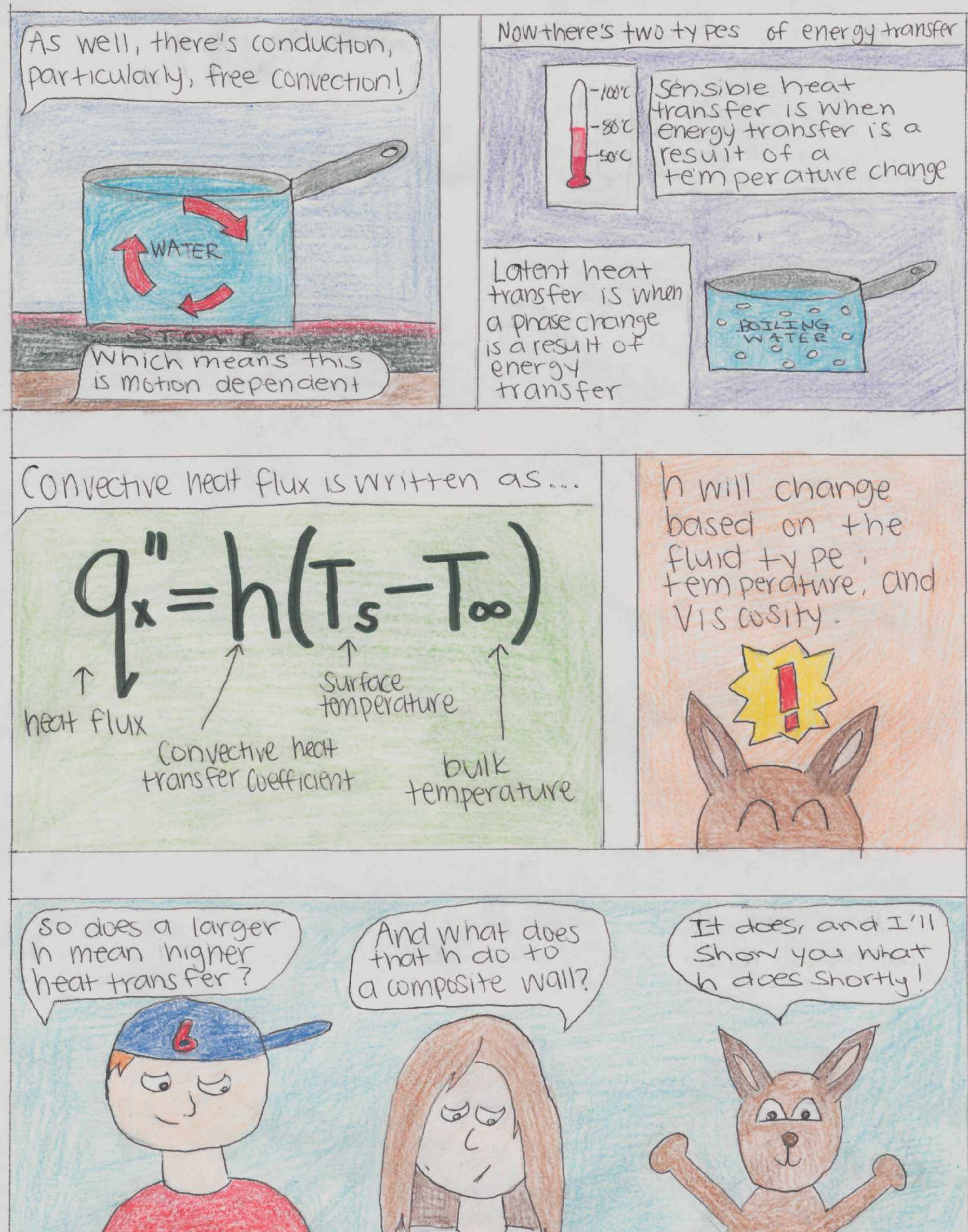
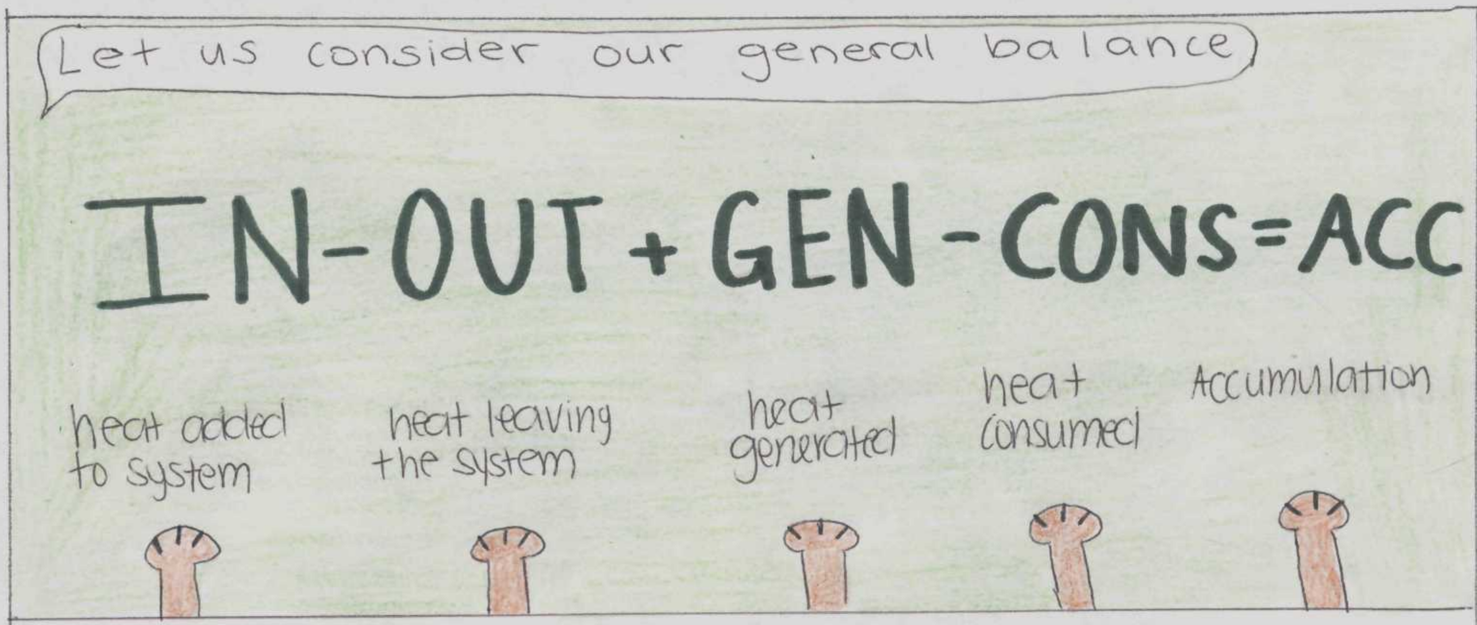
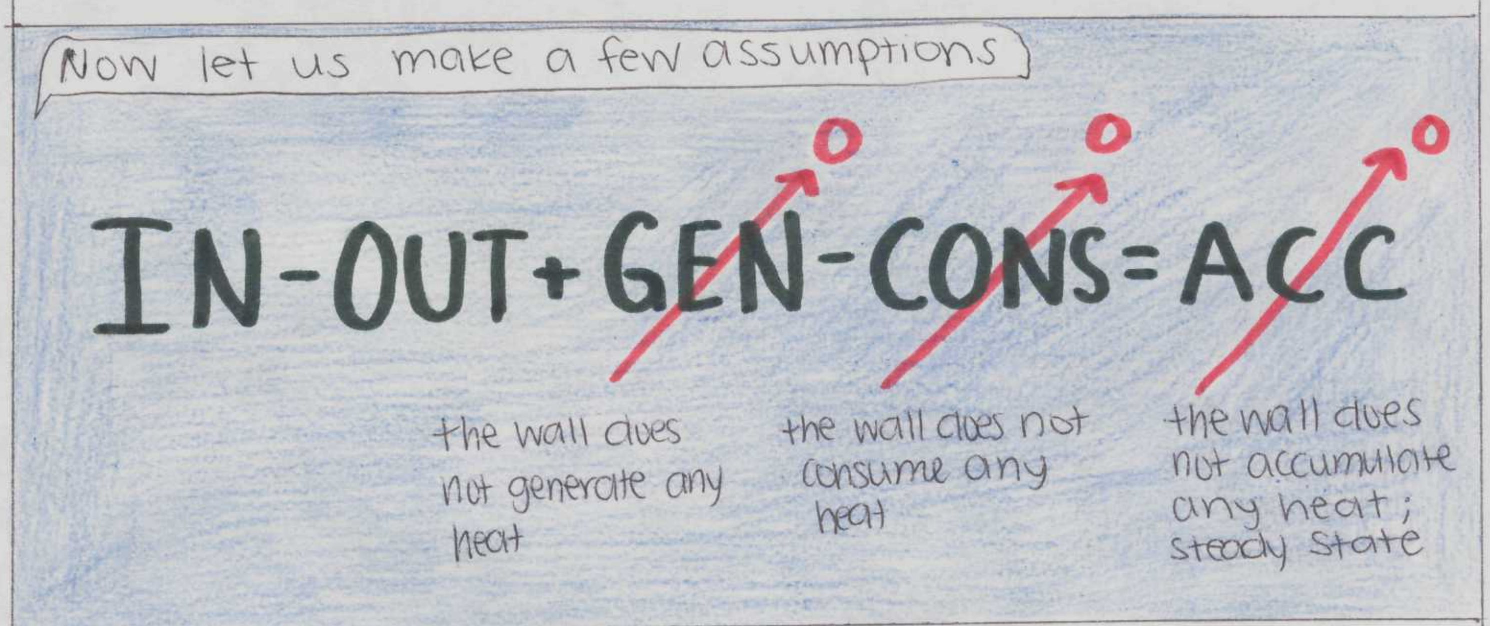
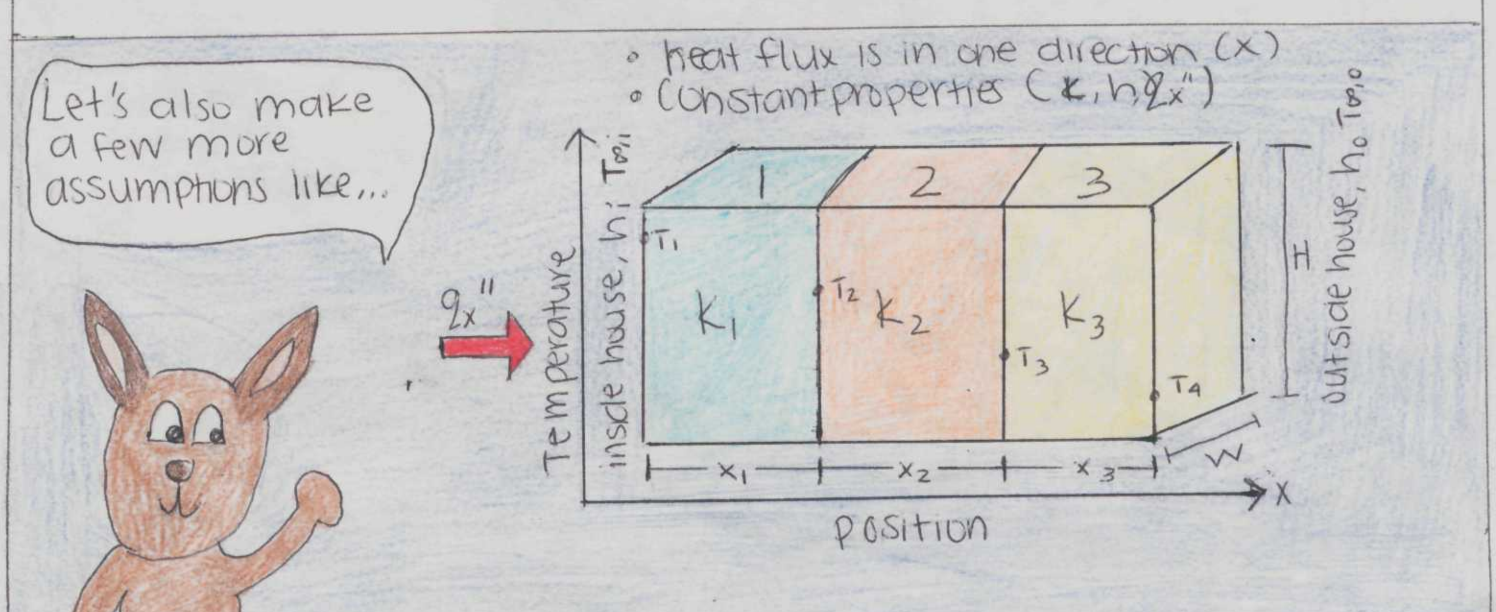
Transport II Course Project

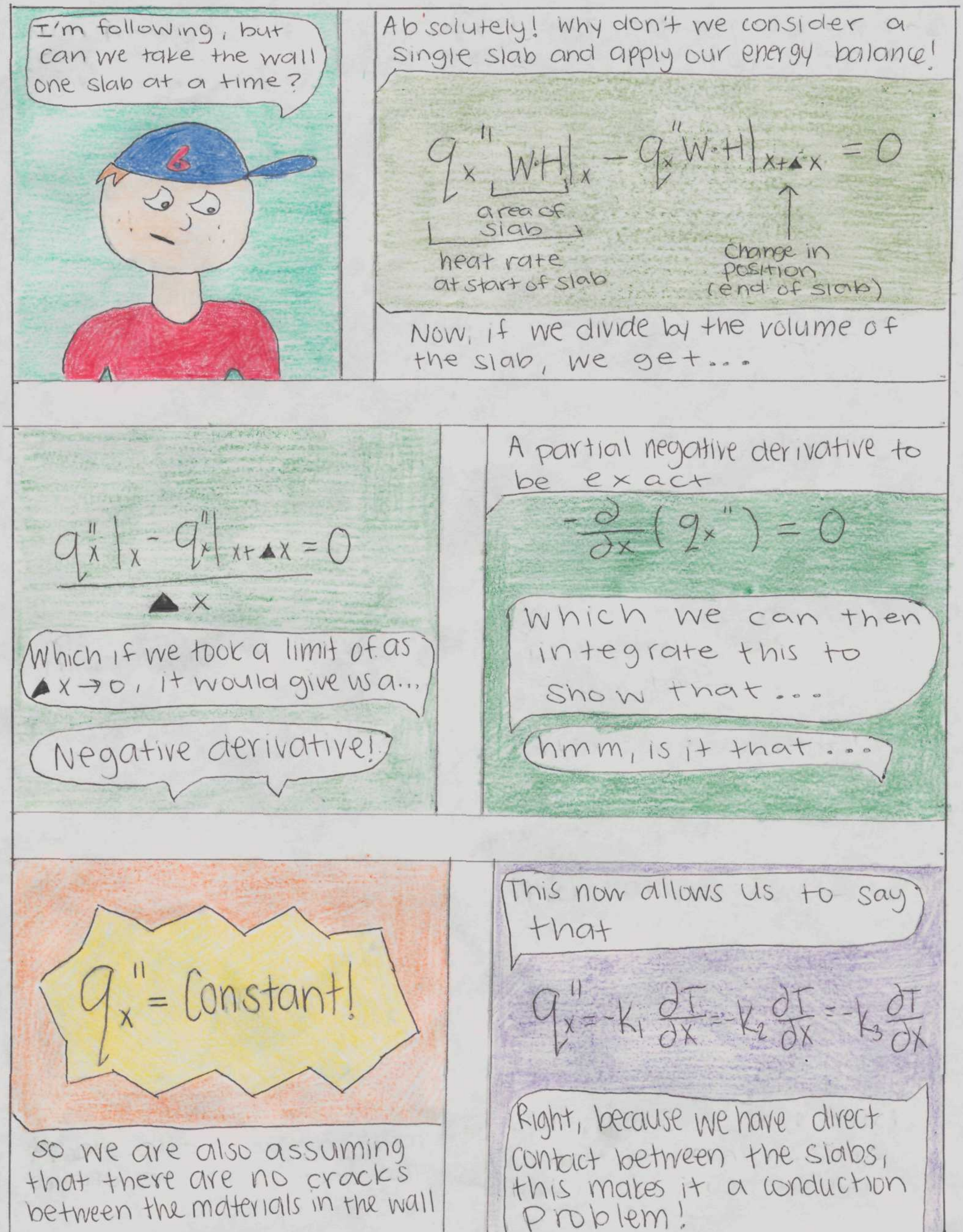

So, let us begin first with some methods of heat transfer!
Well I know conduction is when there is direct contact.
Just like the pot on the stove!
q = Rate of heat flow
HEAT
AREA
q/A = q''
Exactly! Now the rate at which the heat flows through a given area is known as heat flux
Oh, So the flux travels from the stove to the bottom of the pot!
POT
STOVE
Now we're making progress!
This can also be written as Fourier's Law as...
q''x = -K ∂T/∂x
heat flux
heat is transferred in direction of decreasing temperature
thermal conductivity of a material
temperature changes with position
K is important because it will determine how much heat can pass through.
!

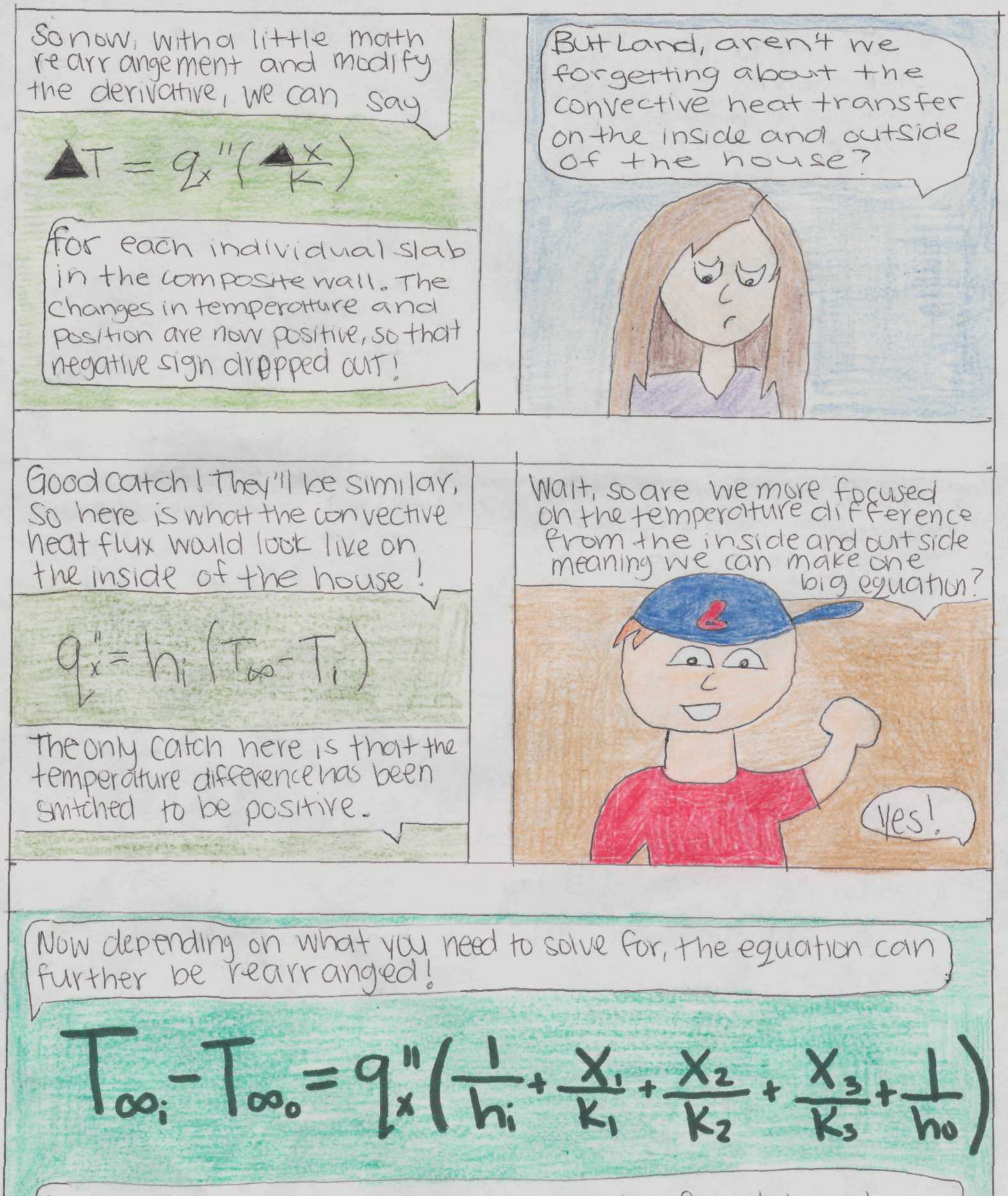

As well, there's conduction, particularly, free convection!
WATER
STOVE
Which means this is motion dependent
Now there's two types of energy transfer
-100°C
-80°C
-50°C
Sensible heat transfer is when energy transfer is a result of a temperature change
Latent heat transfer is when a phase change is a result of energy transfer
BOILING WATER
Convective heat flux is written as...
$q''_x = h(T_s - T_\infty)$
heat flux
convective heat transfer coefficient
surface temperature
bulk temperature
h will change based on the fluid type, temperature, and viscosity.
!
So does a larger h mean higher heat transfer?
And what does that h do to a composite wall?
It does, and I'll show you what h does shortly!

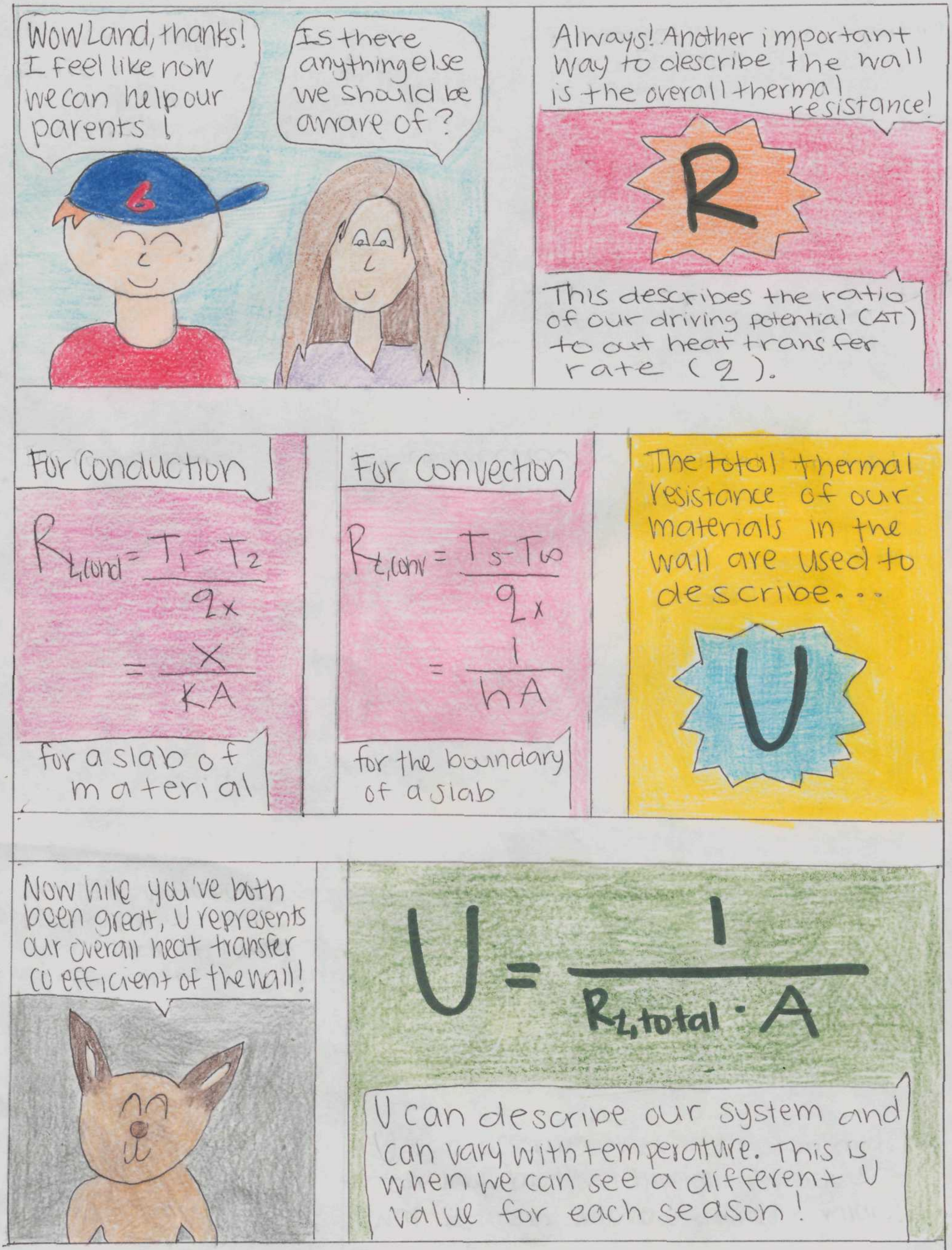

Let us consider our general balance
IN-OUT+GEN-CONS=ACC
heat added to system
heat leaving the system
heat generated
heat consumed
Accumulation


Now let us make a few assumptions
IN-OUT+GEN-CONS=ACC
0
0
0
the wall does not generate any heat
the wall does not consume any heat
the wall does not accumulate any heat; steady state


Let's also make a few more assumptions like,..
• heat flux is in one direction (x)
• Constant properties (k, h, q_x'')
Temperature
inside house, h_i $T_{\infty,i}$
outside house, h_o $T_{\infty,o}$
1
2
3
T_1
T_2
T_3
T_4
k_1
k_2
k_3
q_x''
H
W
x_1
x_2
x_3
x
position


I'm following, but can we take the wall one slab at a time?
Ab'solutely! Why don't we consider a single slab and apply our energy balance!
$q_x'' W \cdot H|_x - q_x'' W \cdot H|_{x+\Delta x} = 0$
area of slab
heat rate at start of slab
Change in position (end of slab)
Now, if we divide by the volume of the slab, we get...
$\frac{q_x''|_x - q_x''|_{x+\Delta x}}{\Delta x} = 0$
Which if we took a limit of as $\Delta x \to 0$, it would give us a...
Negative derivative!
A partial negative derivative to be exact
$-\frac{\partial}{\partial x}(q_x'') = 0$
Which we can then integrate this to show that...
hmm, is it that...
$q_x'' = $ Constant!
So we are also assuming that there are no cracks between the materials in the wall
This now allows us to say that
$q_x'' = -k_1 \frac{\partial T}{\partial x} = -k_2 \frac{\partial T}{\partial x} = -k_3 \frac{\partial T}{\partial x}$
Right, because we have direct contact between the slabs, this makes it a conduction Problem!


So now, with a little math rearrangement and modify the derivative, we can say
$\Delta T = q_x''\left(\frac{\Delta x}{k}\right)$
for each individual slab in the composite wall. The changes in temperature and position are now positive, so that negative sign dropped out!
But Land, aren't we forgetting about the convective heat transfer on the inside and outside of the house?
Good catch! They'll be similar, So here is what the convective heat flux would look like on the inside of the house!
$q_x'' = h_i(T_\infty - T_i)$
The only catch here is that the temperature difference has been switched to be positive.
Wait, so are we more focused on the temperature difference from the inside and outside meaning we can make one big equation?
Yes!
Now depending on what you need to solve for, the equation can further be rearranged!
$T_{\infty_i} - T_{\infty_o} = q_x''\left(\frac{1}{h_i} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} + \frac{1}{h_o}\right)$
Remember, the h and k values can be found based on the materials you use in your composite wall!


WowLand, thanks! I feel like now we can help our parents !
Is there anything else we should be aware of?
Always! Another important way to describe the wall is the overall thermal resistance!
R
This describes the ratio of our driving potential (ΔT) to out heat trans fer rate (q).
For Conduction
$R_{t,cond} = \frac{T_1 - T_2}{q_x} = \frac{x}{kA}$
for a slab of material
For Convection
$R_{t,conv} = \frac{T_s - T_\infty}{q_x} = \frac{1}{hA}$
for the boundary of a slab
The total thermal resistance of our materials in the wall are used to describe...
U
Now hile you've both been great, U represents our overall heat transfer coefficient of the wall!
$U = \frac{1}{R_{t,total} \cdot A}$
U can describe our system and can vary with temperature. This is when we can see a different U value for each season!

And they all lived happily ever after (at steady state)!