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# A failure-dependency modeling and state discretization approach for condition-based maintenance optimization of multi-component systems

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ABSTRACT

Unexpected component failures in a mechanical system always cause loss of performance and functionality of the entire system. Condition-based maintenance decisions for a multi-component mechanical system are challenging because the interdependence of individual components' degradation is not fully understood and lack of physical models. Most existing literature commonly assumes that degradation and failure of individual components within a mechanical system are independent, which could lead to inaccurate diagnostic and prognostic results. In this research, state-rate dependence denoting interaction between component health condition (degradation state) and failure rate is proposed for degradation and failure analysis for a two-component repairable system. A state discretization technique is proposed to model how health state of one component affects the hazard rate of another. An extended proportional hazard model (PHM) is used to characterize the failure dependence and estimate the influence of degradation state of one component on the hazard rate of another. An optimization model is developed to determine the optimal hazard-based threshold for a two-component repairable system. A case study on a generic industrial gearbox has been conducted to show the effectiveness of the proposed model.

## 1. Introduction

Condition-based maintenance (CBM) has received serious attentions in recent years because of improvement in sensor technology and cost reduction in data collection. The majority of CBM research focuses on single-component system. As manufacturing technology advances, manufacturing system becomes more complex and the interrelations among components become more complicated. CBM in complex system becomes very challenging. In addition, the varieties and abundance of the interrelations make CBM in complex system more intractable. As to tackle the problem, the common way is to assume that component degradations or failures are independent. However, this overlooks the truth that stochastic dependence exists, e.g., Bian and Gebraeel [14] and Sun et al. [25] address the genetic gear-box as a typical mechanical system to present the existence of stochastic dependence. In their work, it has been demonstrated that the degradation of a bearing can be reflected from its own vibration amplitude and the vibration can accelerate the degradation of the coupled shaft and other bearings. As a result, the vibrations of the affected bearings increase and exacerbate the degradations and failure rates of other bearings. As to simply deal with failure or degradation dependence among components, researchers assume that the dependence level is defined with prespecified parameters or functions (Li et al. [8]; Hong et al. [9]; Zhang and Yang [18]). Although stochastic dependence exists in multiple forms, extant forms of stochastic dependence can be categorized into four types and presented in Fig. 1. Dependence, such as hazard-hazard dependence (Sun et al. [25]), state-rate dependence (Bian and Gebraeel [14], Rasmekomen and Parlikad [17]), degradation-hazard dependence (Caballé et al. [3]) and shock-degradation dependence (Song et al. [24]) are studied in different multi-component systems. One of the state-rate dependences is state-hazard dependence, which is defined that the degradation state of component influences the hazard rates of other components. State-hazard dependence is rarely studied in CBM.

In this work, an investigation is carried out on how the component states affect the hazard (failure) rate of other components (state-rate dependence) and measuring the magnitude of state-hazard dependence. We develop a method for discretizing component degradation states. We proposed a method for measuring the influence of component state on the hazard rate of other components. A replacement policy taking into account component state information and maintenance cost is developed.

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**Nomenclature**

| | |
|---|---|
| $\Delta$ | inspection interval |
| $\beta$ | shape parameter of Weibull distribution |
| $\gamma$ | weight of covariate |
| $\eta$ | scale parameter of Weibull distribution |
| $r^{(l)}{}_v$ | point index in segment v for observation $l$ |
| $c^{(l)}{}_v$ | number of failures in segment v for observation $l$ |
| $h(t, Z(t))$ | hazard rate given system age $t$ and covariate vector $\mathbf{Z}(t) = [Z_1(t), Z_2(t)]$ |
| $k_{ip}$ | the minimum positive integer number satisfying $kip \le tip$ |
| $n^{(l)}{}_v$ | length of segment $v$ for observation $l$ |
| $t_{ip}$ | the time that the failure risk first reaches the threshold given state vector $\mathbf{Z}(t) = [i, p]$ |
| $C_{AC}$ | average cost per unit time |
| $C_{cycle}$ | total cost in association with repair and replacement per cycle |
| $C_m(k,i,p)$ | expected repair cost due to failure given component age $k$ and covariates $i$ and $p$ |
| $C_{me}$ | cost of per minimal repair performed on component $e$ |
| $C_p$ | total cost of replacing component 1 and 2 |
| $D$, $d$ | condition-based threshold |
| $N(D)$ | number of failure repairs before replacement |
| $\boldsymbol{P}$ | covariate state transition probability matrix |
| $R(k,i,p,t-k\Delta)$ | conditional reliability function until time t given the age of the system is $k\Delta$ and $\mathbf{Z}(k\Delta) = [i,p]$ |
| $S_c$ | contrast function optimization stopping criteria |
| $T_r$ | replacement time; |
| $U(V)$ | contrast function; |
| $V$ | number of change-points |
| $W(D)$ | expected replacement time |
| $W(k,i,p)$ | expected replacement time given state vector $\mathbf{Z}(t) = [i, p]$ |
| $Z_{FT}$ | physics-based threshold (fault threshold) |
| $Z(t)$ | value of the stochastic covariate at time $t$ |
| $\mathbf{Z}(t)$ | covariate vector $\mathbf{Z}(t) = [Z_1(t), Z_2(t)]$ contains state covariates of components 1 and 2 at time $t$ |
| $\Vert\hat{\Sigma}\Vert$ | determinant of empirical covariance matrix $\hat{\Sigma}$ |

## 2. Background literature

Review works in CMB from different perspectives are presented by researchers. Cho and Parlar [6] survey the literature on maintenance and replacement models for multi-component system. A number of models, such as repair model, group/opportunistic model, maintenance/replacement model, and inspection/maintenance model are presented. Jardine et al. [1] review machinery diagnostics and prognostics implementing CBM. The overview synthesizes the processes from degradation data acquisition and data analysis until maintenance decision-making. Techniques for dealing with different data forms, such as time-domain data, frequency-domain data and value type data, are mentioned. Alaswad and Xiang [22] present a panoramic view of CBM for single-component system. Their review work focuses on inspection performance, maintenance quality and maintenance optimization criteria. The demand of CMB for multi-component system with stochastic dependence has been emphasized.

### *2.1. CBM for single-component system*

Numerous papers are published on CBM for single-component system (Banjevic et al. [4]; Banjevic and Jardine [5]; Shafiee et al. [16]; Peng et al. [19]; Vlok et al. [20]; Makis and Jardine [27]; Zhu et al. [28]). Banjevic et al. [4] propose a model known as control limit policy for maintenance decision-making. An iteration algorithm and a recursive procedure are developed in the proposed model as to obtain the optimal preventive replacement threshold. Based on the control limit policy, the research is further extended by Banjevic and Jardine [5] for remaining useful life estimation. Peng et al. [19] present their research on a single-component system suffering multiple dependent competing failure processes. The studied failure processes are competing and deemed as interdependent.

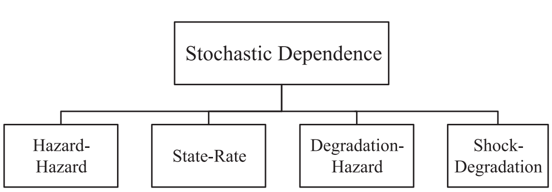


**Fig. 1.** Type of stochastic dependence.

### *2.2. CBM for multi-component system*

Because of the importance and demand of CBM for multi-component system, an abundance of research investigating different forms of dependence has been published. In contrast to stochastic dependence, economic dependence is easy to manage and studied in plentiful research (Bouvard et al. [12]; Tian and Liao [30]; Tian et al. [32]). CBM in multi-component system with stochastic dependence is few due to the variety and complexity of stochastic dependence. As to make the maintenance modeling of multi-component system simple, Zhu et al. [29] assume that the studied failure modes, hard failure and soft failure, are independent. The maintenance performance with imperfect prediction signal is investigated. In fact, the assumption, that degradation processes of components in complex system are independent, is lack of justification and always results in errors in estimating system reliability or lifetime.

Golmakani and Moakedi [10] develop a model to find out optimal inspection interval for a two-component repairable system with failure interaction. Failures are classified into soft and hard, and hard failure has influencing effect on soft failure and can not affected by soft failure. Song et al. [24] propose a new reliability model for a series system subject to competing hard and soft failure processes. Shocks can cause hard failure and incremental progress on soft failure processes. Rasmekomen and Parlikad [17] propose a model with state-rate interactions. They aim at identifying the optimal inspection timing and preventive replacement threshold for each tube under multiple maintenance strategies. Zhang et al. [31] develop a mathematical model by taking into account opportunistic maintenance and environmental influence to determine an optimal maintenance policy for a multi-component system. The environmental conditions are shown to exert an influence on the component degradation processes. Caballé et al. [3] propose a CBM strategy for the system subject to two dependent causes of failure: degradation processes and sudden shocks. The studied deterioration levels of the degradation processes directly influence the sudden shock process and indirectly affect the intensity of total failure of the system. In order to study how stochastic dependence level influences maintenance strategy, copulas (Li et al. [8]; Hong et al. [9]; Zhang and Yang [18]), such as Levy copula, Gumbel copula, Clayton copula and normal copula, are proposed to model the magnitude of stochastic dependence. Stochastic dependence with different magnitudes is investigated via the marginal distribution functions (Li et al. [8]; Hong et al. [9]). A dependent latent age model for capturing reliabilities of components with multiple competing failure modes and failure interdependence is developed by Zhang and Yang [18]. A joint distribution is constructed via copula functions for characterizing interdependence of component failures.

As to dynamically utilize degradation information, Horenbeek and Pintelon [2] present a dynamic predictive maintenance policy by accommodating short-term component degradation information. Their intention is to predict component's remaining useful life (RUL) via dynamically adapting the maintenance planning. The interactions between degradation rate and degradation state are studied in Bian and Gebraeel [14]. They propose a Bayesian framework to dynamically update information associated with component residual lifetime distributions via degradation interaction analysis. In their article, they assume that component degradation rate is influenced by the degradation states of other components and the degradation interactions characterize with linear functions.

### 2.3. Proportional hazard model for single-component system

Prior to demonstrating the proposed methodologies for multi-component system, a model from previous research work for single-component system is presented in this section. Cox and Oake's [7] *proportional hazard model* (PHM) with baseline Weibull hazard function and time-dependent stochastic covariates is used to describe equipment deterioration process. The equipment failure behavior can be reflected from the equipment age and state. The hazard function representing equipment hazard rate is expressed as follows:

$$h(t, Z(t)) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[\sum_{g=1}^{m} \gamma_g Z_g(t)\right] \tag{1}$$

$\beta$ and $\eta$ are the shape and scale parameters of the Weibull distribution; $h_0(s) = \beta/\eta(t/\eta)^{\beta-1}$ is the baseline hazard rate taking into account the age of the equipment, $t$ is the equipment age; covariate vector $\boldsymbol{Z}(t) = (Z_1(t), Z_2(t), \ldots, Z_m(t))$ represents covariate processes, $\exp[\sum_{i=1}^{m} \gamma_g Z_g(t)]$ takes into account equipment health condition or operating environment. Covariate $Z_g(t)$ can be classified as internal or external based on the definition provided by Kalbfleisch and Prentice [14]. Internal covariates indicate the health condition of component. External covariates, such as temperature, pressure, vibration, can affect the degradation process.

A cost model (Banjevic et al. [4]; Makis and Jardine [27]) is developed to obtain the optimal preventive replacement threshold $d$. The expected average cost per unit time incurred from preventive replacement is shown:

$$\begin{aligned} C_{wins} &= \frac{C(1 - P(T \le T_d)) + (C + K)P(T \le T_d)}{E(\min\{T, T_d\})} \\ &= \frac{C(1 - Q(d)) + (C + K)Q(d)}{W(d)} \end{aligned} \tag{2}$$

where $C_{wins}$ is the average cost per unit time; $C$ is the preventive replacement cost per replacement; $C + K$ is the failure replacement cost per replacement; $d$ is the threshold for preventive replacement; $P(T \le T_d)$ and $Q(d)$ equivalently denote the probability that failure replacement takes place before equipment failure rate reaches the threshold $d$; $(1 - Q(d))$ is the probability that component survives until preventive replacement; $E(min\{T, T_d\})$ and $W(d)$ equivalently denote the expected time until replacement, regardless of preventive replacement or failure replacement. In order to compute $Q(d)$ and $W(d)$, a form of recursive computation procedure is developed in Makis and Jardine [27].

### 2.4. Paper outline

The model in Section 2.3 shows the uniqueness in obtaining a risk-based threshold for single-component system. In this work, we use the PHM to investigate failure interactions between component states and failure rates in a repairable multi-component system. Components within the repairable system mutually affects each other. We develop a novel method to segment the continuous covariates, which indicate component health state or operating environment, into discrete states. By taking into account the discretized states, we develop a cost model in association with minimal repair and replacement to optimize the condition-based replacement threshold. To implement the proposed model, the genetic gear-box as a typical series system is studied for demonstration. Analytical results in the case study show that component hazard rates can significantly be reflected from the discrete states. Additionally, the sensitivity analysis is performed to show how the replacement policy changes.

The paper is organized as follows. Section 3 comprises the proposed state discretization technique and the model to derive the optimal replacement threshold. In Section 4, a case study demonstrates the algorithm of the proposed state discretization technique and acquisition of the optimal replacement threshold. A set of sensitivity analysis on maintenance policy is presented. Section 5 concludes our work and addresses future research directions.

## 3. Proposed CBM for multi-component system

We extend the PHM model discussed in Section 2.3 from the single-component system to the multi-component system. It is shown in Jardine et al. [1] and Bian and Gebraeel [14] that the degradation state of one component can accelerate the degradation process and lifetime of other components, vice versa. Because randomness of failure is often involved in the degradation process (Jardine et al. [1]). Our proposed technique integrates both degradation and event (failure) data jointly and the assumptions for our work are given below:

- All the components are subject to monotonic non-decreasing degradation, component hazard rate is a function of time and the observed internal and external covariates;
- Failure can happen at any instant, component failure makes the system stop;
- Minimal repair can restore system full functionality and does not affect the state of the repaired system. Replacement is perfect and restores the system to be as-good-as-new, repair and replacement time are negligible;
- Condition monitoring is continuous and failure can only be detected at the inspection. Inspection cost is negligible.

### 3.1. Covariate state discretization

The main idea of the proposed state discretization technique is to segment continuous degradation state into discrete bands by integrating component degradation data and failure data. The segmented bands correspond to discrete states indicating component health condition. The component in different states has different hazard rates. The boundaries for discretizing continuous states into discrete bands are in correspondence with the changes in the combination of degradation state and hazard rate of the component. The proposed state discretization method is based on change-point detection technique. Different change-point detection techniques have been presented (Schwarz [23]; Lavielle and Teyssiere [15]; Guralnik and Srivastava [26]). Schwarz criteria developed by Schwarz [8] is commonly used for model selection and change detection. Schwarz criteria has numerous applications, such as model selection in ecology and evolution (Jahnson et al. [11]) and stock indices returns and financial market (Lavielle and Teyssiere [15]). In Lavielle and Teyssiere [15], the multiple change-point problems for multivariate time series with different level of dependences are investigated. An adaptive method is proposed and applied to multivariate series data from stock and financial market. Guralnik and Srivastava [26] propose two algorithms, batch algorithm and incremental algorithm, for event detection from time series data. Our proposed state discretization method is based on Schwarz [23] and Lavielle and Teyssiere [15]. Detailed explanation of the proposed algorithm is further provided in this section.

In this article, the studied genetic gear-box consists of input shaft bearing and output shaft bearing. The degradation state of input shaft bearing can accelerate the degradation process and failure process of the output shaft bearing. Correspondingly, the affected output shaft bearing generates analogous effects on the input shaft bearing. This indicates the two bearings are mutually influential and affected. Let $e$ denote the index of the bearing, here $e = 1, 2$. Let vector $\mathbf{Z}(t)$ be the system condition and operating environment. We consider single covariate for each bearing, hence $\mathbf{Z}(t) = [Z_1(t), Z_2(t)]$. $Z_1(t)$ and $Z_2(t)]$ are continuous stochastic process that can influence component failure time (Makis and Jardine [27]). Here $Z_1(t)$ indicates the degradation state of input shaft bearing and $Z_2(t)$ indicates the degradation state of output shaft bearing. The covariate of one bearing is considered internal when indicating the bearing degradation state and considered external when it affects the failure rate of another bearing. Researchers have showed that time-continuous covariates $Z(t)$ can be segmented into discrete states (Banjevic et al. [4] ; Banjevic and Jardine [5]; Bian and Gebraeel [14]). Accordingly, component has different degradation or hazard rates in different states. As to measure the effects of the discretized states on bearing hazard rate, we develop a novel method for discretizing continuous covariates into segmented states. State discretization for both bearings is shown in Fig. 2.

The circle in the coordinate denotes the detected faults or failures, at which minimal repair is performed. The triangle in the upper coordinate corresponding to times $\{\theta_1, \theta_2, \theta_3\}$ at which crossing time $t = \theta_v$ bearing 1 (input shaft bearing) has significant change of its degradation rate and hazard rate. The changes are caused by either the state of component itself or state of another component. The proposed algorithm integrates degradation and failure data together and the changes are the consequence of the analytical combination of degradation and failure data. $\theta_v$ is the $v^{th}$ change point. $\{Z(\theta_1), Z(\theta_2), Z(\theta_3)\}$ denotes the observed degradation states of bearing 1 at times $\{\theta_1, \theta_2, \theta_3\}$. Consequently, continuous covariate $Z_1(t)$ is discretized into multiple states $\{s_{11}, s_{12}, s_{33}, s_{44}\}$. $Z_1(t)$ is assumed to be constant within each segment. Likewise, the lower coordinate shows the observed degradation states $\{Z_2(\theta_1), Z_2(\theta_2), Z_2(\theta_3)\}$ for bearing 2 (output shaft bearing) given the same times $\{\theta_1, \theta_2, \theta_3\}$. Because the proposed covariate discretization technique integrates degradation data and failure data together. The time-lag between a failure and the change point $\theta_v$ comes from the integration of degradation rate change and hazard rate change jointly. Let integer numbers stand for the discretized $Z_e(t)$, hence $\{s_{11} = 0, s_{12} = 1, s_{13} = 2,\ldots, s_{1s} = s_1\}$ for bearing 1 and $\{s_{21} = 0, s_{22} = 1, s_{23} = 2,\ldots, s_{2s} = s_2\}$ for bearing 2. $s_{e1} = 0$ indicates component in the best state and $s_{es} = s_e$ indicates the worst state. The key idea is to identify the threshold $\boldsymbol{f} = \{f_{e,1}, f_{e,2}, \ldots, f_{e,V}\}$ where bearings have abrupt degradation rate shift and hazard rate change. A state discretization algorithm is developed and presented in the followings.

### 3.1.1. Step 1. Identify optimal number of change points

The first step is to identify the optimal number of change points. Let $t^{(l)}max$ denote the maximum time index at which both bearings are replaced for experiment $l$. The total number of time points is $n^{(l)}$ within $t^{(l)}max$ and $n^{(l)} = t^{(l)}max$. As to identify the optimal number of change points $V^*$, the contrast function, which measures the quality of number of segments is given below:

$$U(V)=\sum_{l=1}^{N}\left[\frac{1}{n^{(l)}}\sum_{v=1}^{V} n^{(l)}{}_v \log\|\hat{\Sigma}_{\tau^{(l)}{}_v}\| + V\left[\left(V+1\right)c^{(l)}{}_1 + Vc^{(l)}{}_2 + \ldots + c^{(l)}{}_{v+1}\right]\right] \tag{3}$$

where $n^{(l)}$ is the time length of experiment $l$. $n^{(l)}{}_v = \tau^{(l)}{}_v - \tau^{(l)}{}_{v-1}$ is the length of segment $v$ for observation $l$. $c^{(l)}{}_v$ is the number of failures in segmentation $v$, include failures of bearing 1 and bearing 2. In practice, a large number of states will increase our calculation complexity and difficulty. In addition, increasing the number of discrete states does not significantly improve our proposed model in obtaining the optimal maintenance cost. Therefore $V[(V+1)c^{(l)}_1 + Vc^{(l)}_2 + \ldots + c^{(l)}_{v+1}]$ serves as the penalty function in case that the obtained optimal $V^*$ is too large. The empirical covariance matrix $\hat{\Sigma}_{\tau^{(l)}_v}$ computed on segment $v$ for record

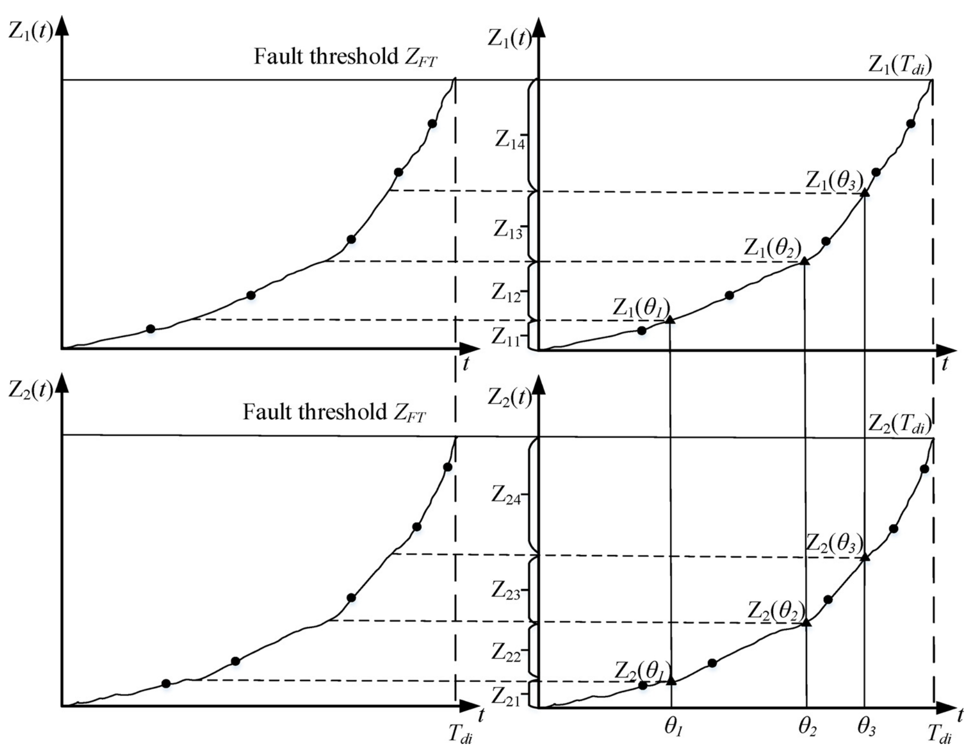


**Fig. 2.** Covariate state discretization.

$l$ is:

$$\hat{\Sigma}_{\tau_v^{(l)}} = \frac{1}{n_v^{(l)}} \sum_{t=\tau_{v-1}^{(l)}+1}^{\tau_v^{(l)}} (Z_t - \overline{Z}_{\tau_v^{(l)}})(Z_t - \overline{Z}_{\tau_v^{(l)}}) \tag{4}$$

Let $V$ start with 1 until the largest $Vmax$ is reached. By increasingly assigning a value to $V$, Eq. (3) can be used to derive the corresponding change points $\{\theta^{(l)}{}_{e1}, \theta^{(l)}{}_{e2}, \ldots, \theta^{(l)}{}_{eV}\}$ for record $l$. Let the user-defined stability threshold stopping criteria $S_c$ equal to 0. The optimal $V^*$ is identified from an iteration procedure when the stopping criteria $S_c$ is met:

$$\frac{U(V) - U(V+1)}{U(V)} < S_c \tag{5}$$

The corresponding change points (times) $\{\theta^{(l)}{}_{e1}, \theta^{(l)}{}_{e2}, \ldots, \theta^{(l)}{}_{eV^*}\}$ are obtained. We assume that the component condition is observable and measureable at any time. Hence for record $l$, its covariate threshold $\mathbf{Z}_e^{(l)} = \{Z^{(l)}{}_{e1}, Z^{(l)}{}_{e2}, \ldots, Z^{(l)}{}_{eV^*}\}$ of bearing $e$ can be obtained. As to generalize the thresholds for the dataset with $N$ records, the threshold $\mathbf{Z}_e^{(l)} = \{Z^{(l)}{}_{e1}, Z^{(l)}{}_{e2}, \ldots, Z^{(l)}{}_{eV^*}\}$ for each record is exploited in step 2.

### 3.1.2. Step 2. Identify threshold $\boldsymbol{f} = \{f_{e,1}, f_{e,2}, \ldots, f_{e,V^*}\}$ of covariate state discretization for each component

In step 1, change points $\{\theta^{(l)}{}_{e1}, \theta^{(l)}{}_{e2}, \ldots, \theta^{(l)}{}_{eV^*}\}$ and covariate threshold $\mathbf{Z}_e^{(l)} = \{Z^{(l)}{}_{e1}, Z^{(l)}{}_{e2}, \ldots, Z^{(l)}{}_{eV^*}\}$ of component $e$ for record $l$ are identified. Based on the historical data with $N$ records, the threshold $\boldsymbol{f}_e = \{f_{e,1}, f_{e,2}, \ldots, f_{e,V^*}\}$ for component $e$ is expressed as follows:

$$\boldsymbol{f}_e = \frac{\boldsymbol{Z}_e^{(1)} + \boldsymbol{Z}_e^{(2)} + \cdots + \boldsymbol{Z}_e^{(N)}}{N} \tag{6}$$

Covariate state $Z_e(t)$ is piecewise constant and a constant number within each discretized segments $[f_{e,1}, f_{e,2})$. The piecewise constant numbers indicating component health condition for each segments is shown:

$$Z_e(t) = \begin{cases} 0 & 0 \le Z_e(t) < f_{e,1} \\ 1 & f_{e,2} \le Z_e(t) < f_{e,3} \\ & \vdots \\ V^* & f_{e,V^*} \le Z_e(t) \le f_{e,V^*+1} \end{cases} \tag{7}$$

### 3.1.3. Step 3. Discretized state transition probability

This step is focused on estimating transition process $Z_e(t)$ between discretized segments. State space for component $e$ is $S_e = \{0, 1, 2, \ldots, V^*\}$ and $Z_e(t) \in S_e$. Banjevic et al. [2] develop a maximum likelihood method for estimating the transition probability between states. Let $p_{ij}(t) = P(Z_{k+1} = j | T > t,\ Z_k = i)$ denote the transition probability from state $i$ to state $j$ at time $t$; $i, j \in S_e$. $T$ denotes the failure time here. Let $\boldsymbol{P}^{(1)}$ and $\boldsymbol{P}^{(2)}$ be the estimated state transition matrixes for bearing 1 and bearing 2. State transition matrix $\boldsymbol{P}_{ij,pm}$ for the two-bearing system is:

$$\boldsymbol{P}_{ij,pm} = (\boldsymbol{P}^{(1)})_{ij} \cdot (\boldsymbol{P}^{(2)})_{pm} \tag{8}$$

## 3.2. Condition-based replacement policy optimization

After obtaining the discretized covariate states, this section aims at estimating the parameter of the hazard model for each bearing. Let vector $\boldsymbol{h}(\boldsymbol{t}) = (h_1(t, \mathbf{Z}(t)), h_2(t, \mathbf{Z}(t)))$ denote the hazard rate of bearing 1 and 2 at time $t$, here vector $\mathbf{Z}(t) = [Z_1(t), Z_2(t)]$ contains state covariates of bearing 1 and 2. Let vector $\boldsymbol{t} = (t_1, t_2)$ denote the age of bearing 1 and 2. The reliability for bearing 1 at age $t_1$ is as follows:

$$P(T_1 > t_1) = R_1(t_1, Z(t_1)) = \exp\left\{-\int_0^{t_1} h(s, Z(s))ds\right\} \tag{9}$$

where $T_1$ denotes the failure time of bearing 1. Likewise, the reliability of bearing 2 is given $P(T_2 > t_2) = R_2(t_2, Z(t_2)) = \exp\left\{-\int_0^{t_2} h(s, Z(s))ds\right\}$.

We assume the replacement is performed on both bearings and restores the system to be as-good-as-new, hence $t_1 = t_2$. Let $t$ denote the system age, $t = t_1 = t_2$. Because the studied genetic gearbox is a series system and the system reliability is expressed as follows:

$$\begin{aligned} P(T > t) &= R(t, Z(t)) = P_1(T > t)P_2(T > t) = R_1(t, Z(t))R_2(t, Z(t)) \\ &= \exp\left\{-\int_0^t h_1(s, Z(s))ds\right\}\exp\left\{-\int_0^t h_2(s, Z(s))ds\right\} \\ &= \exp\left\{-\int_0^t [h_1(s, Z(s)) + h_2(s, Z(s))]ds\right\} \end{aligned} \tag{10}$$

Eq. (10) shows that the hazard rate for the series system is the summation of hazard rate of all components. Let $h(t,\mathbf{Z}(t))$ be the hazard rate of the system, $h(t,\mathbf{Z}(t)) = h_1(t,\mathbf{Z}(t)) + h_2(t,\mathbf{Z}(t))$. We define a threshold $D$, the replacement is performed on both components whenever system hazard rate $h(t,\mathbf{Z}(t))$ reaches the threshold $D$. We assume a basic inspection interval $\Delta$ and component state $Z(k\Delta)$ is only observable at discrete time $t = k\Delta$, where $k = 0, 1, 2, 3, \ldots$. Bearing failure is random and can occur at any instant before the system hazard rate $h(k\Delta,\mathbf{Z}(k\Delta))$ reaches the threshold $D$. Let $k_1$ and $k_2$ denote the bearing age index. Let $k$ denote system age index, here $k = k_1 = k_2$. The system reliability within interval $[k\Delta, (k+1)\Delta]$ is:

$$\begin{aligned} R(k, Z(k\Delta)) &= P_1(T > k\Delta)P_2(T > k\Delta) = R_1(k, Z(k\Delta))R_2(k, Z(k\Delta)) \\ &= \exp\left\{-\exp(\gamma_{11}Z_1 + \gamma_{12}Z_2)\int_{k\Delta}^{(k+1)\Delta} h_{10}(s)ds \right. \\ &\quad \left. - \exp(\gamma_{21}Z_1 + \gamma_{22}Z_2)\int_{k\Delta}^{(k+1)\Delta} h_{20}(s)ds\right\} \end{aligned} \tag{11}$$

When k = 0, all the covariates are in the best states $\mathbf{Z}(0) = [(0,0), (0,0)]$. Therefore, the process between successive replacements is a regenerative process. The total cost $C_{cycle}$ in association with repair and replacement for each regenerative cycle is:

$$C_{cycle} = N_1C_{m1} + N_2C_{m2} + C_p = C_m + C_p \tag{12}$$

where $N_1$ and $N_2$ are the number of repairs on component 1 and 2, $C_{m1}$ and $C_{m2}$ are the corresponding repair costs for bearing 1 and 2 per repair. $C_m$ denotes the total repair cost including repair cost for both bearing 1 and 2. $C_p$ denotes the replacement cost for replacing bearing 1 and 2. $T_r$ denotes the replacement time. Because the degradation process of both bearings after time $T_r$ is independent of bearing states before $T_r$. The average cost per unit time can be obtained for each cycle. Let $E[T_r]$ be the expected time between two successive replacements. The average cost per unit time $C_{AC}$ is expressed as follows:

$$C_{AC} = \frac{C_{cycle}}{E[T_r]} = \frac{N_1(D)C_{m1} + N_2(D)C_{m2} + C_p}{W(D)} = \frac{C_m(D) + C_p}{W(D)} \tag{13}$$

In order to calculate $C_{AC}$, $C_m(D)$ and $W(D)$ are derived from following equations. Given $t_{ip} = inf\{t \ge 0 | h_1(t,i,p) + h_2(t,i,p) \ge D\}$, here $i$ and $p$ are the state of bearing 1 and 2 as Section 3.1 demonstrates, and $k_{ip} \le t_{ip} < k_{ip} + 1$. Let $R_1(k,i,p,t)$ denote the reliability of bearing 1 until time t given system age is k and $\mathbf{Z}(t) = [i, p]$. Similarly, let $R_2(k,i,p,t)$ be the reliability of bearing 2. Let $C_m(k, i, p)$ be the expected repair cost due to failure given the system age $k$ and states $i$ and $p$. When $k > k_{ip}$, $C_m(k, i, p) = 0$. When $k = k_{ip}$

$$\begin{aligned} C_m(k, i, p) &= C_{m1}[1 - R_1(k, i, p, t_{ip} - k\Delta)]R_2(k, i, p, t_{ip} - k\Delta) \\ &\quad + C_{m2}[1 - R_2(k, i, p, t_{ip} - k\Delta)]R_1(k, i, p, t_{ip} - k\Delta) \\ &= C_{m1}C_{m2}[1 - R_1(k, i, p, t_{ip} - k\Delta)][1 - R_2(k, i, p, t_{ip} - k\Delta)] \end{aligned} \tag{14}$$

when $k < k_{ip}$

$$\begin{aligned} C_m(k, i, p) &= C_{m1}[1 - R_1(k, i, p, \Delta)]R_2(k, i, p, \Delta) \\ &\quad + C_{m2}[1 - R_2(k, i, p, \Delta)]R_1(k, i, p, \Delta) \\ &= C_{m1}C_{m2}[1 - R_1(k, i, p, \Delta)][1 - R_2(k, i, p, \Delta)] \\ &\quad + \sum_{j=i}^{V^*}\sum_{m=p}^{V^*} P_{ij,pm}C_m(k+1, j, m) \end{aligned} \tag{15}$$

The expected repair cost for each cycle $C_m(D) = C_m(0,0,0)$. The next objective is to obtain $E[T_r]$. Let W($k$,$i$,$p$) denote the expected replacement time given $\mathbf{Z}(t) = [i, p]$ and the system age $k$. When $k > k_{ip}$, W($k$,$i$,$p$) = 0. When $k = k_{ip}$

$$W(k, i, p) = t_{ip} - k_{ip} \tag{16}$$

When $k < k_{ip}$

$$W(k, i, p) = \Delta + \sum_{j=i}^{V^*}\sum_{m=p}^{V^*} P_{ij,pm}W(k+1, j, m) \tag{17}$$

The expected replacement time $W(D)$ = W(0,0,0), W(0,0,0) can be obtained.

As to find out the optimal threshold, a small value is initially assigned to threshold $D$ and the calculated $C_{AC}$ is very large due to frequent replacement. Next, we iteratively increase the threshold $D$. The corresponding $C_{AC}$ increases when $D$ is small and starts increasing when the threshold $D$ is getting large. After crossing a specific threshold $D$, the calculated $C_{AC}$ starts increasing. The specific threshold $D$ is optimal and the calculated $C_{AC}$ is minimum. With the help of Matlab, the developed optimization algorithm is implemented (Fig. 3).

The flowchart above illustrates the procedure of our proposed

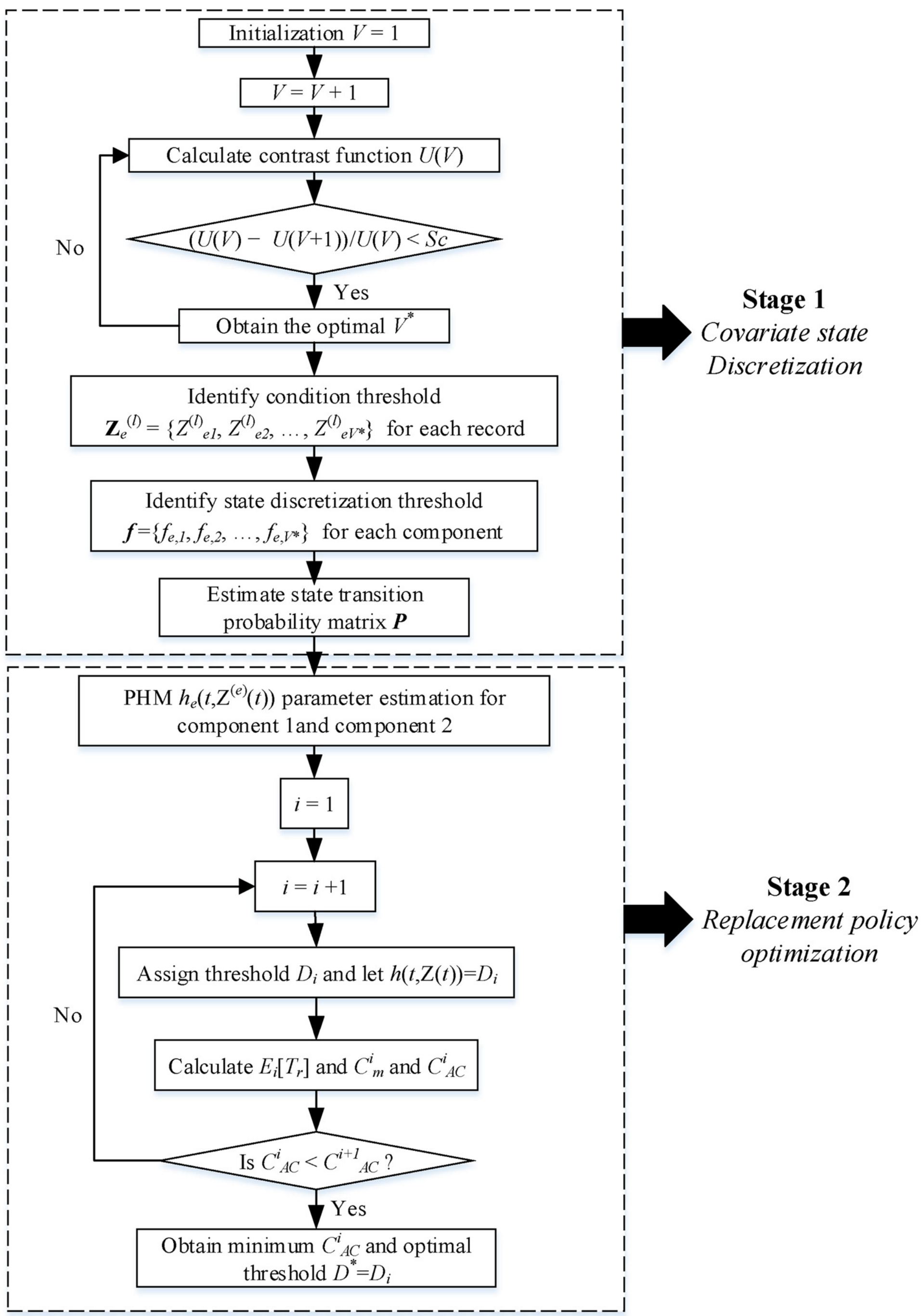


**Fig. 3.** Flow chart to determine optimal replacement policy.

model. It consists of two stages. Stage 1 is aimed to discretize continuous covariate state into discrete states. Stage 2 is targeted at optimizing the replacement policy.

## 4. Case study

We demonstrate the proposed method using simulated data representing the generic industrial gearbox system with two roller bearings that are subjected to vibration monitoring. We simulate the roller bearing degradation process by taking into account failure randomness and physical dependence between the two roller bearings. We set the inspection interval ($\Delta$) as 20 days. The run time of the simulation is set as ($100\Delta$) 2000 days. We set the state threshold $Z_{FT}$ equal to 200. The system is deemed to be failed whenever either/both bearing reach the threshold before the run time 2000 days. The failed system needs replacement for both bearings. Each instance generates a pair of vibration measurement. Ten pairs of vibration measurements are generated and the underlying degradation process for each instance is extracted (see Fig. 4).

The following assumptions are considered in the simulation environment: a) When a failure occurs, a minimal repair can restore the full system functionality and it does not change the bearing degradation state, b) the repair cost for each bearing is fixed, c) after each replacement maintenance (maintenance undertaken to replace the bearing), the system is assumed to be in same state as the state representing the new system, and d) the replacement cost is constant. Considering the aforementioned simulation environment along with its assumptions, the objective is to optimize the condition-based replacement policy to minimize the total costs associated with repair and replacement of the component.

$t^{(l)}max$ is the time when both bearings are replacement for record $l$, hence the maximum of $t^{(l)}max$ is 100. Based on the Eqs. (3)–(6), the optimal number of segments $V^* = 3$ and the discretized covariate state ranges for bearing 1 are [0, 47.8), [47.8, 87.6), [87.6, 136.9) and [136.9, 185]. Eq. (7) shows that the covariate state is constant within each range. Hence $\boldsymbol{Z}_1 \in \{Z_{11} = 0, Z_{12} = 1, Z_{13} = 2, Z_{14} = 3\}$. Similarly, the covariate state ranges for bearing 2 are [0, 44.5), [44.5, 82.2), [82.2, 138.5) and [138.5, 185], $\boldsymbol{Z}_2 \in \{Z_{21} = 0, Z_{22} = 1, Z_{23} = 2, Z_{24} = 3\}$. The estimated state transition probability for bearing 1 and 2 are given in Tables 1 and 2.

**Table 1**
Discretized covariate state range and transition matrix for bearing 1.

| Covariate state | Covariate range | 0<br>[0, 47.8) | 1<br>[47.8, 87.6) | 2<br>[87.6, 136.9) | 3<br>[136.9, 185] |
|---|---|---|---|---|---|
| 0 | [0, 47.8) | 0.8231 | 0.1231 | 0.0538 | 0.0000 |
| 1 | [47.8, 87.6) | 0.0000 | 0.8478 | 0.1231 | 0.0291 |
| 2 | [87.6, 136.9) | 0.0000 | 0.0000 | 0.8185 | 0.1815 |
| 3 | [136.9, 185] | 0.0000 | 0.0000 | 0.0000 | 1.0000 |

**Table 2**
Discretized covariate state range and transition matrix for bearing 2.

| Covariate state | Covariate range | 0<br>[0, 44.5) | 1<br>[44.5, 82.2) | 2<br>[82.2, 138.5) | 3<br>[138.5, 185] |
|---|---|---|---|---|---|
| 0 | [0, 44.5] | 0.8197 | 0.1286 | 0.0517 | 0.0000 |
| 1 | [44.5, 82.2) | 0.0000 | 0.8122 | 0.1424 | 0.0000 |
| 2 | [82.2, 138.5) | 0.0000 | 0.0000 | 0.7946 | 0.2054 |
| 3 | [138.5, 185] | 0.0000 | 0.0000 | 0.0000 | 1.0000 |

The estimated hazard rates for both bearings are $h_1(t) = (9.1752/50.3601)(t/50.3601)^{8.1752}\ exp[1.8022Z_1(t) + 1.8144Z_2(t)]$ and $h_2(t) = (7.9783/53.0260)(t/53.0260)^{6.9783} exp[1.8082Z_1(t) + 1.5493Z_1(t)]$. The coefficient of covariate $Z_2(t)$ in $h_1(t)$ is 1.8022, which indicates that the state (covariate) of bearing 2 accelerates the degradation and failure rate of bearing 1. Likewise, the coefficient of covariate $Z_1(t)$ in $h_2(t)$ is 1.8082 indicating that the state (covariate) of bearing 1 accelerates the degradation and failure rate of bearing 2. Both processes $\{Z_1(t), t > 0\}$ and $\{Z_2(t), t > 0\}$ have four states. Interaction term of $Z_1(t)$ and $Z_2(t)$ is not considered. The estimation shows strong significance of the time $t$, which conforms to the nature of degradation in the mechanical system. The estimated coefficients for the covariates are significant (small P-value) and indicates that the covariates significantly influence the hazard rate of the bearing.

### 4.1. Condition-based replacement policy optimization

Once the model parameters are obtained by the proposed method,

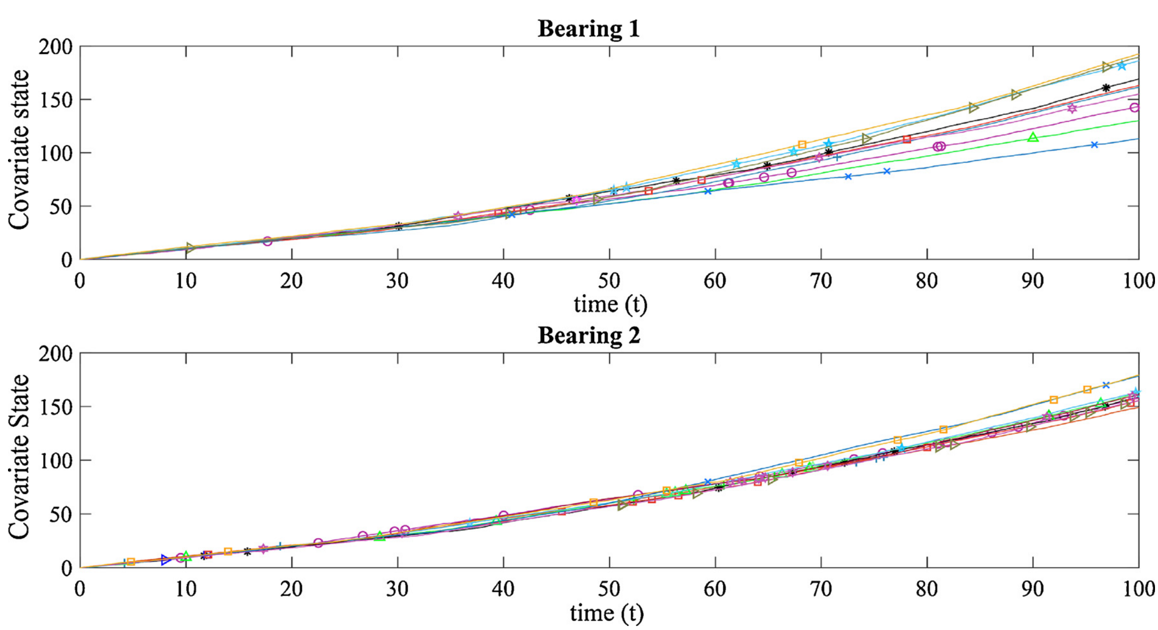


**Fig. 4.** Extracted roller bearing degradation process from the simulated data. The line in the figure denotes the roller bearing degradation process. Points (circle, plus sign, diamond, etc.) on the line denote the failures.

the next step is aimed at obtaining the optimal threshold $D^*$ to minimize the cost associated with failure repair and replacement. Repair costs bearing 1 are set as $C_{m1}$ = \$600 and for bearing 2 are set as $C_{m2}$ = \$800. The replacement cost for replacing both components is $C_p$ = \$12,000. Based on the maintenance policy, repair maintenance is performed when a component fails (either bearing 1 or 2). Here we assume that the repair on one component does not affect the performance of the other component. The hazard rate of the system is denoted as $h(t,\mathbf{Z}(t))$. When system hazard rate reaches $h(t,\mathbf{Z}(t)) \geq D$, replacement operation is performed on both components. After the replacement operation, the system state represents the same system sate as new. With the optimization tool in MATLAB and the aforementioned assumptions, the minimum costs associated with repair and replacement is \$715.377/period and \$35.76885/day. The corresponding optimal expected replacement time $E[T_r]$ = 18.6024 periods (372.048 days) and optimal condition-based threshold $D^*$ = 1.21.

The small threshold $D$ keeps the system in operation with high reliability. Hence it incurs less repair cost and reduces the frequency of replacement. Fig. 5 shows the relation between threshold $D$ and the expected replacement time $E[T_r]$. We observe that $E[T_r]$ increases with an increase in threshold $D$. When $D$ is large, system has longer replacement cycle time and increasing $D$ affects a small increment in $E[T_r]$. When threshold $D$ is small, $E[T_r]$ sensitivity to $D$ increases greatly.

Average cost $C_{AC}$ is associated with repair and replacement and threshold $D$. For a fixed threshold $D$, relation between total cost $C_T$ and replacement cost $Cp$ is linear. Fig. 6 shows repair cost per unit time $C_m/\mathrm{E}[T_r]$, replacement cost per unit time $C_p/\mathrm{E}[T_r]$ and average cost $C_{AC}$ by changing threshold $D$. As the blue line shows, $C_m/\mathrm{E}[T_r]$ increases as threshold $D$ increases. On the contrary, On the contrary, an increasing threshold $D$ reduces the replacement cost per unit time $C_p/\mathrm{E}[T_r]$. Relatively large average cost $C_{AC}$ is obtained when $D$ is small. Initially small increments in $D$ are followed by a decreasing $C_{AC}$ due to the lower cost incurred from the repair. However after crossing a specific threshold $D$ (optimal threshold $D^*$), $C_{AC}$ starts increasing due to higher repair cost. A threshold $D$ value either too small or too large generates a large $C_{AC}$ value. A large threshold $D$ results in expenditure incurred from frequent replacement. A small threshold $D$ requires more repair and correspondingly incurs more cost. Minimum cost $C_{AC}$ can only be obtained when threshold $D$ is neither too small nor too large. The sudden jumps of the curves in Figs. 5 and 6 come from the state transition. Because component state transition can occur at any time index $k$. The component may be in the same state or transition to a severe state from current time index $k$ to the next time index $k + 1$.

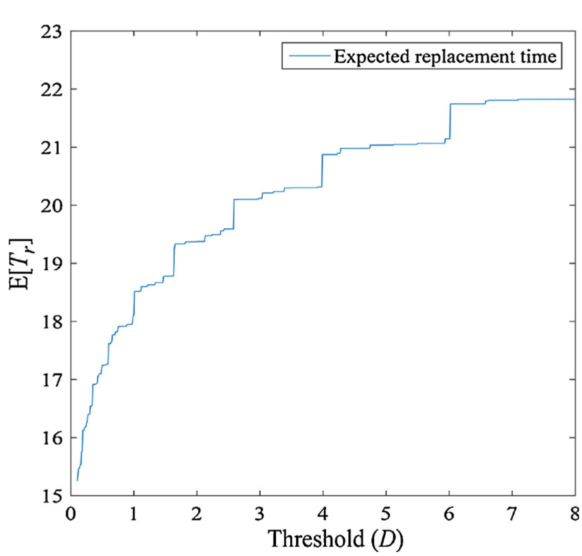


**Fig. 5.** Threshold $D$ vs replacement time $E[T_r]$.

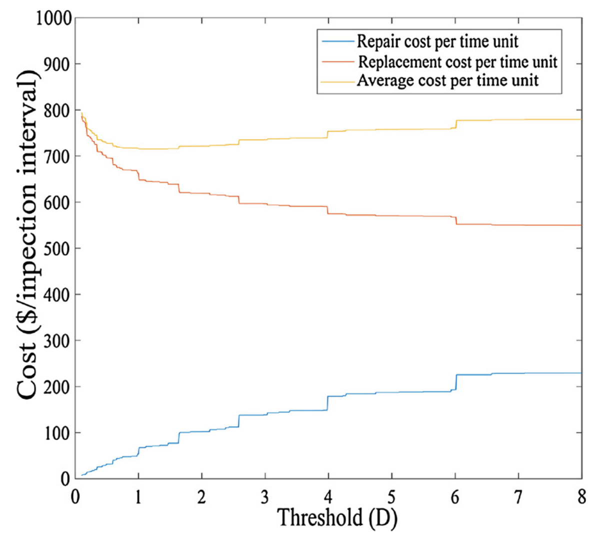


**Fig. 6.** Threshold $D$ vs costs.

### 4.2. Sensitivity analysis

The following sensitivity analysis aims to investigate how the maintenance policy changes in terms of the variations in the repair cost, replacement cost and the level of dependency. The respective optimal cost measure and the optimal threshold $D^*$ are examined accordingly.

#### 4.2.1. Repair and replacement cost analysis

We first perform sensitivity analysis of the repair costs and the threshold under two scenarios. In scenario 1, we set $C_p$ = \$12,000 and $C_{m2}$ = \$800, by changing $C_{m1}$ from \$200 with increment \$200 to \$2000. In scenario 2, we set $C_p$ = \$1200 and $C_{m1}$ = \$50, by changing $C_{m2}$ from \$200 with increment \$200 to \$2000.

Fig. 7(a) for scenario 1 shows that threshold $D^*$ decreases as repair cost $C_{m1}$ increases. It shows that optimal threshold $D^*$ highly increases by increasing $C_{m1}$ in the range [\$400, \$1000]. As repair cost $C_{m1}$ becomes large, increment in $C_{m1}$ receives small deduction in optimal threshold $D^*$. Fig. 7(b) for scenario 2 shows that optimal threshold $D^*$ decreases as repair cost $C_{m2}$ increases. Likewise, the reduction in threshold $D^*$ is significant while $C_{m2}$ is in the range [\$400, \$1000]. The increment of $C_{m2}$ results in insignificant decrease in optimal threshold $D^*$. Meanwhile, Fig. 8 shows how the repair costs $C_{m1}$ and $C_{m2}$ influence threshold $D^*$ below.

Fig. 8 shows that the optimal threshold $D^*$ decreases when $C_{m1}$ or/and $C_{m2}$ increases and $C_p$ is fixed. The flat blue areas stand for large repair costs $C_{m1}$, $C_{m2}$ and small optimal threshold $D^*$. Repair costs, either $C_{m1}$ or $C_{m2}$ is large or $C_{m1}$ and $C_{m2}$ both are large, the optimal threshold $D^*$ barely changes when $C_{m1}$ changes or $C_{m2}$ changes or $C_{m1}$ and $C_{m2}$ both change. The observed results demonstrate that the optimal threshold $D^*$ is sensitive to some specific ranges and $C_{m2}$. This observation is also confirmed by the results shown in Fig. 7(a) and (b).

After the sensitivity analysis of repair costs is conducted. We first perform the sensitivity analysis of the replacement cost to investigate how the minimum average cost per unit time $C_{AC}$ and the optimal threshold $D^*$ change. We set $C_{m1}$ = \$600 and $C_{m2}$ = \$800 and change $C_p$ from \$8000 to \$20,000.

Fig. 9(a) shows a positive linear relationship between the replacement cost $C_p$ and the minimum cost $C_{AC}$. It shows that the minimum cost $C_{AC}$ linearly increases as $C_p$ increases. The minimum cost $C_{AC}$ is very small when the replacement cost $C_p$ is small. Fig. 9(b) shows that the optimal threshold $D^*$ is non-decreasing while increasing $C_p$. This is because that replacement incurs too much cost and is not expected to perform frequently, hence a large $D^*$ is desired when $C_p$ is large. The results demonstrate that the optimal threshold $D^*$ is sensitive to some

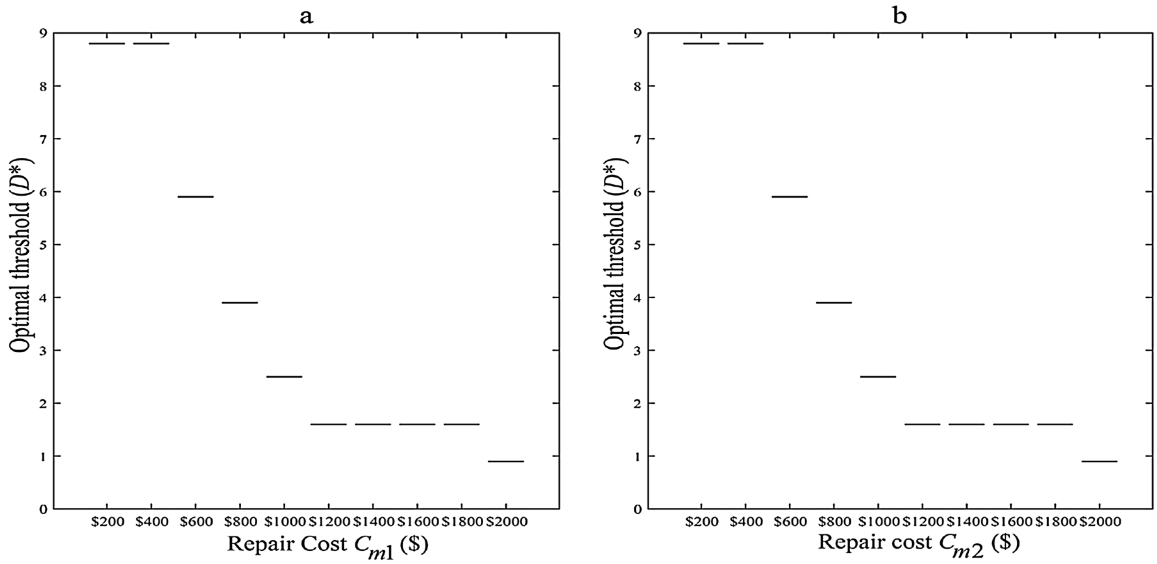


**Fig. 7.** Repair cost vs optimal threshold $D^*$. (a) $C_{m1}$; (b) $C_{m2}$.

specific ranges when $C_p$ is small. It also appears the optimal threshold $D^*$ remains unchanged when $C_p$ is large enough.

*4.2.2. Level of dependence analysis*

To investigate the changes in the optimal threshold $D^*$ and the minimum cost $C_{AC}$ under the various levels of dependence. Two scenarios are designed. In scenario 1, we fix the shape and scale parameters of $h_1(t)$ of bearing 1. Meanwhile, the covariate coefficient of $h_1(t)$ $\gamma_{11}$ is changed from 0 to 2 and $\gamma_{12}$ is changed from 0 to 2. The value of $\gamma_{11}$ means that bearing 1 has different hazard rate in different states. $\gamma_{11} = 0$ indicates that the hazard rate $h_1(t)$ has no difference when bearing 1 is in different states. $\gamma_{11} = 2$ indicates that $h_1(t)$ has salient difference and increases as the state of bearing 1 progresses. The value of $\gamma_{12}$ indicates the magnitude of effect from bearing 2 on $h_1(t)$. $\gamma_{12} = 0$ represents that bearing 2 has no effect on the hazard rate of bearing 1 and $\gamma_{12} = 2$ indicates that the state of bearing 2 excessively accelerates the hazard rate of bearing 1. In scenario 2, we also fix the shape and scale parameters of $h_2(t)$ and set covariate coefficients $\gamma_{21}$ from 0 to 2 and $\gamma_{22}$ from 0 to 2. The value of $\gamma_{12}$ indicates the magnitude of effect from bearing 1. $\gamma_{21} = 0$ represents that the state of bearing 1 neither accelerates nor deaccelerates $h_2(t)$. $\gamma_{21} = 2$ indicates that the state of bearing 1 severely accelerates the failure rate of bearing 2. Likewise, the value of $\gamma_{22}$ means that bearing 2 has different hazard rate in different states. $\gamma_{22} = 0$ indicates that the hazard rate $h_2(t)$ has no difference even though bearing 2 is in different states. $\gamma_{22} = 2$ indicates that $h_2(t)$ has salient difference and increases as the state of bearing 2 transitions to a progressed state.

Given the designed scenarios, the optimal threshold $D^*$ and the minimum cost $C_{AC}$ are obtained and showed in Fig. 10. Fig. 10(a) represents the trend of optimal threshold $D^*$ under different combinations of $\gamma_{11}$ and $\gamma_{12}$. It shows the optimal threshold $D^*$ is not predictable and no specific pattern can be found from Fig. 10(a). The minimum cost $C_{AC}$ appears in Fig. 10(c) representing that $C_{AC}$ increases as $\gamma_{11}$ and/or $\gamma_{12}$ increases. When both $\gamma_{11}$ and $\gamma_{12}$ are close to 2, $C_{AC}$ is very sensitive and increase significantly. $C_{AC}$ shows less sensitivity when $\gamma_{11}$ and $\gamma_{12}$ are comparably small.

Unlike the optimal threshold $D^*$ in scenario 1, Fig. 10(b) shows that the optimal threshold $D^*$ is relatively constant when $\gamma_{21}$ is small and $\gamma_{22}$ is large. $D^*$ is very sensitive when both $\gamma_{21}$ and $\gamma_{22}$ are large. The minimum cost $C_{AC}$ appears in Fig. 10(d) representing the same pattern as $C_{AC}$ in scenario 1. $C_{AC}$ barely changes when $\gamma_{11}$ and $\gamma_{12}$ are small. When both $\gamma_{11}$ and $\gamma_{12}$ are close to 2, $C_{AC}$ is very sensitive and increase significantly.

*4.3. Comparative study*

In this section, the independence assumption is used as the benchmark in comparison with our proposed approach. The implementation

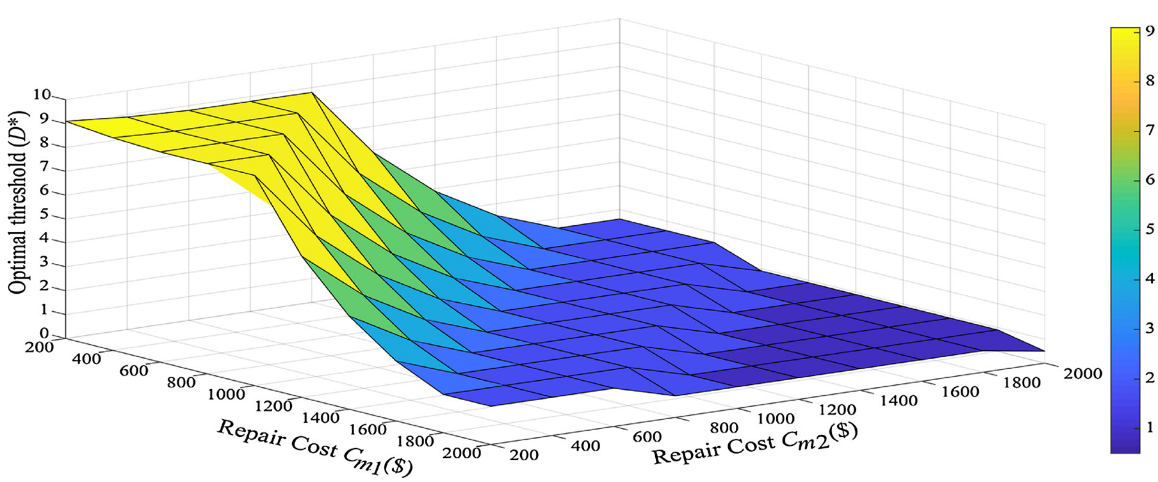


**Fig. 8.** Effect of repair costs $C_{m1}$ and $C_{m2}$ on optimal threshold $D^*$.

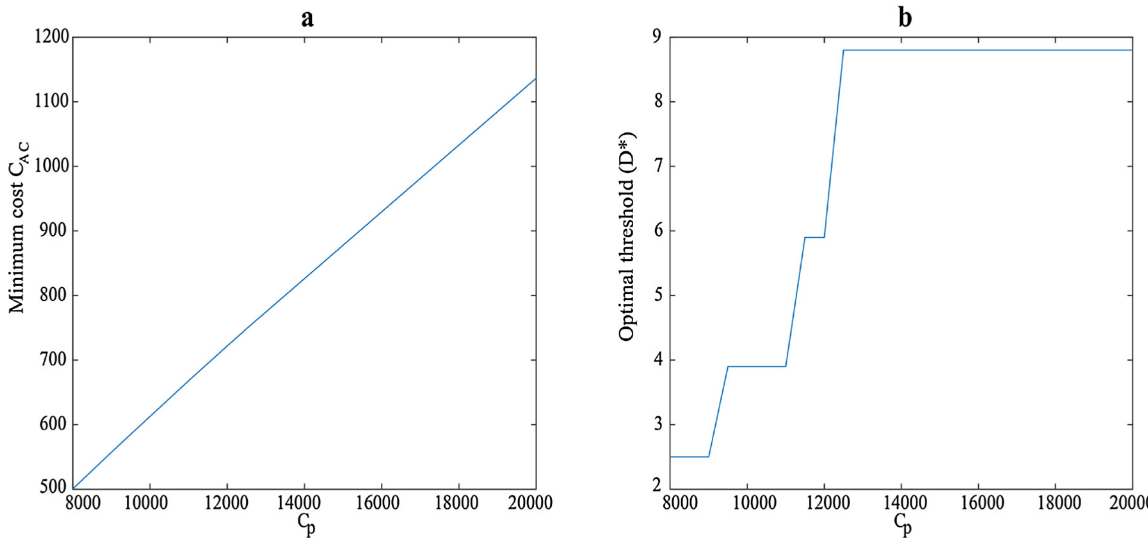


**Fig. 9.** Effect of replacement cost $C_p$ on the minimum cost $C_{AC}$ and optimal threshold $D^*$.

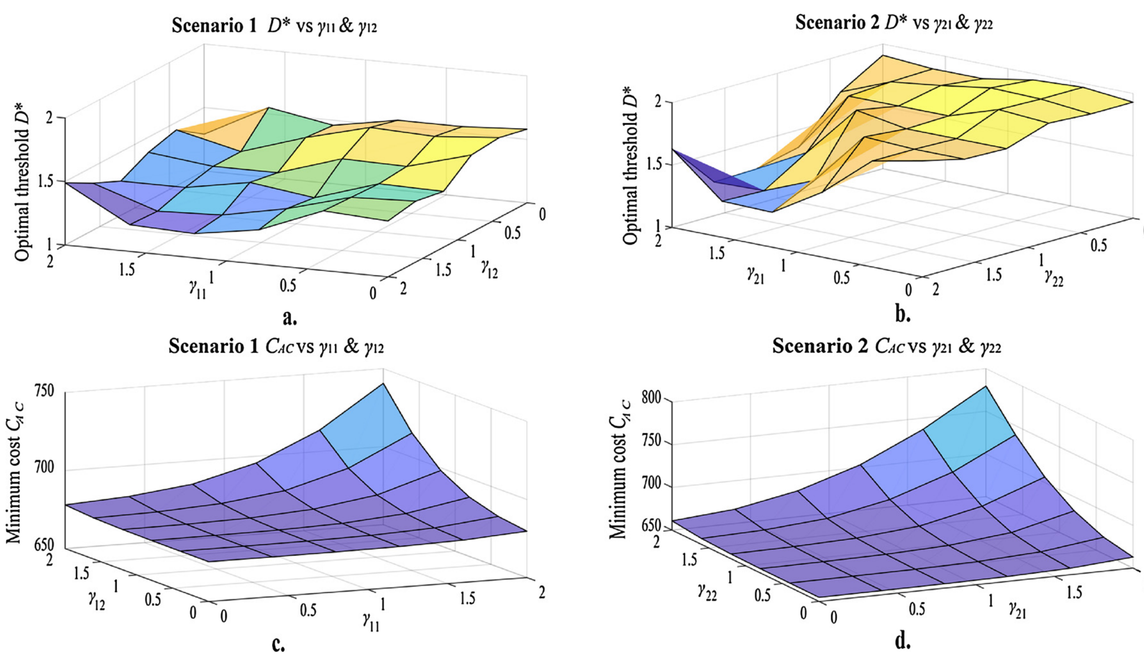


**Fig. 10.** Effect of level of dependence on the optimal threshold $D^*$ and minimum $C_{AC}$.

**Table 3**
Goodness-of-fit analysis.

| | Model | Shape $\beta$ | Scale $\eta$ | Covariates | | AIC | BIC |
|---|---|---|---|---|---|---|---|
| | | | | Bearing 1 | Bearing 2 | | |
| Independent | $h_1(t)$ | 4.3172 | 41.9384 | 0.7330 | – | 455.41 | 457.64 |
| | $h_2(t)$ | 3.5369 | 41.6694 | – | 0.6245 | 462.96 | 465.18 |
| Interdependent | $h_1(t)$ | 9.1752 | 50.3601 | 1.8022 | 1.8144 | 364.10 | 365.18 |
| | $h_2(t)$ | 7.9783 | 53.0260 | 1.8082 | 1.5493 | 357.14 | 361.58 |

of comparison is completed from the perspectives of the goodness of fit test and cost analysis.

### 4.3.1. *Data modeling analysis*

A comparative study between the independence assumption and our proposed model from the perspective of goodness-of-fit is completed. As to fit the PHM of each bearing under the independence assumption, we assume that the hazard rate of the bearing is not affected by the state of another component. Therefore, the PHM model of each bearing is a function of the age and internal covariate state indicating its own health condition. By using technical standards, the four covariate states are established (Banjevic et al. [4]). In order to test the goodness-of-fit, *Akaike information criterion* (AIC) [21] and *Bayesian information criterion* (BIC) [23], which are the most common methods for testing the quality

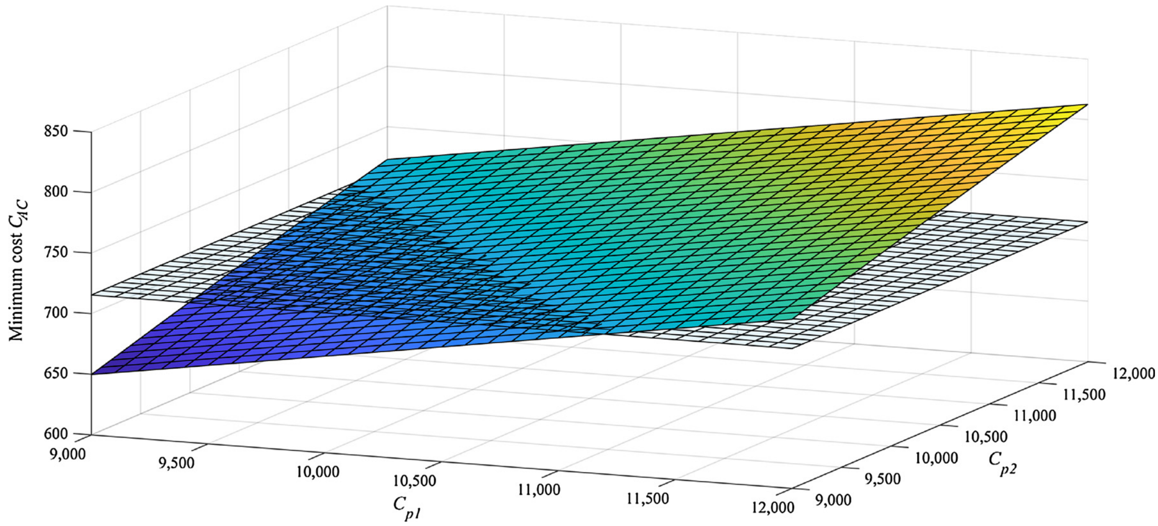


**Fig. 11.** Effect of $C_{p1}$ and $C_{p2}$ on minimum cost $C_{AC}$ under independence assumption.

of statistical models and the goodness-of-fit for model selection, are used as the metrics to evaluate the fitted model. The results from the test appear in Table 3.

Under the independence assumption, the hazard rate $h_1(t)$ of bearing 1 is a function of time $t$ and its own covariate state. Likewise, the hazard rate $h_2(t)$ of bearing 2 is a function of time $t$ and its own covariate state. The estimated parameters of $h_1(t)$ and $h_2(t)$ show at the first row and second row following 'Independent' in Table 3. Under the interdependence assumption, the estimated parameters of $h_1(t)$ and $h_2(t)$ shown at the beginning of Section 4 show at the first row and second row following 'Interdependent' in Table 3. The AIC and BIC at the last two columns show that the AIC and BIC under the independence assumption are larger, which indicates the goodness-of-fit of PHM under the independence assumption are inferior compared with our proposed technique. Hence, independence assumption that degradation of one bearing is not affected by the states of other bearing is lack of justification and cause the fitted model lack of accuracy.

### 4.3.2. Cost analysis

Under the independence assumption, the desired threshold for replacement is component-level. Let $D_1$ and $D_2$ denote the condition-based threshold for triggering replacement on bearing 1 and 2. $C_{p1}$ and $C_{p2}$ denote the replacement costs of bearing 1 and 2. The average costs per unit time of component 1 and 2 are $C^{(1)}{}_{AC}$ and $C^{(2)}{}_{AC}$. Thus, the total average cost per time unit $C_{AC} = C^{(1)}{}_{AC} + C^{(2)}{}_{AC}$. Because group replacement has been introduced into our model and $C_p$ is the cost of replacing both bearing 1 and bearing 2 at the same time, we assume $C_{p1} + C_{p2} > C_p$ based on the fact that group maintenance is more cost-effective than individual maintenance. The objective of this section is to investigate how the difference between $C_{p1} + C_{p2}$ and $C_p$ affects $C_{AC}$ under the assumption of independence.

Let $C_p$ = \$12,000 for replacing both bearings and the repair costs for bearings 1 and 2 are $C_{m1}$ = \$600 and $C_{m2}$ = \$800 respectively as introduced in Section 4.1. With our proposed approach, the derived minimum $C_{AC}$ is \$715.377/period and is represented with the flat plane (black-white color) in Fig. 11. Under the independence assumption, repair costs for bearing 1 and 2 are as same as $C_{m1}$ and $C_{m2}$ used under our proposed technique. We change the replacement cost $C_{p1}$ of bearing 1 from \$9000 to \$12,000. Meanwhile, we change the replacement cost $C_{p2}$ of bearing 2 from \$9000 to \$12,000. The colored plane in Fig. 11 denotes the minimum total cost $C_{AC}$ under the independence assumption. It shows that $C_{AC}$ increases when $C_{p1}$ or/and $C_{p2}$ increases. In comparison with $C_{AC}$ obtained from our proposed technique, the total cost $C_{AC}$ under the independence assumption is less when the difference between $C_{p1} + C_{p2}$ and $C_p$ is small. Replacement costs, either $C_{p1}$ or $C_{p2}$ is large or $C_{p1}$ and $C_{p2}$ both are large (the difference between $C_{p1} + C_{p2}$ and $C_p$ is large), the minimum total cost $C_{AC}$ under the independence assumption increases significantly, and the obtain $C_{AC}$ is larger than the $C_{AC}$ obtained from our proposed method. On the other hand, when the difference between replacement cost of group maintenance and individual maintenance gets larger, then the derived maintenance policy by considering failure dependence is found to be more cost-effective.

## 5. Conclusions and future work

CBM in a complex mechanical system is challenging due to interdependence of degradation behavior among components within the system. Most existing literature commonly assumes that degradations and failures between components within a mechanical system are independent or the level of degradation dependence is pre-specified. Such assumptions could cause ineffective degradation analysis and inaccurate condition assessment. In this research, state-rate dependence denoting interaction between component health condition (degradation state) and failure rate is proposed for degradation and failure analysis for a two-component repairable system. A state discretization technique is proposed to model how the health state of one component affects the hazard rate of another. The PHM is extended from the single-component system to the multiple-component system for characterizing the failure dependence and estimating the influence of degradation state on the hazard rate. An optimization model is developed to derive the optimal hazard-based threshold for a two-component repairable system. An experimental study is used to demonstrate effectiveness of the proposed model.

The proposed discretization technique can capture the degradation dependence between components and guarantee the accuracy of estimating component reliability. The condition-based threshold determined by the proposed model for replacement decision is system-level and can obtain the optimal CMB policy corresponding to the minimum cost by taking advantage of group replacement. However, when the system-level hazard rate reaches the replacement threshold, all the components in the multi-component system will be replaced. This may cause component underutilization if one of the components is still in good condition. The component-level threshold by considering degradation interdependence is more accurate and computationally expensive. A computationally lean algorithm is needed for determining the component-level risk-based threshold for the multi-component system. In addition, the performance of minimal repair is not studied. For example, minimal repair may not restore the full functionality of the system. The effect of minimal repair may make the proposed policy different and increase the minimum average maintenance cost. Furthermore, the proposed model is only applied to a two-component repairable system. Extending the proposed model to a complex system with three or more components may decay its accuracy. Future research will extend the proposed model to a complex system with three or more components.

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