The Attribution of The Unknown Remainder to God

“Every number is known to Him whose understanding cannot be numbered. Although the infinite series of numbers cannot be numbered, this infinity is not outside His comprehension. It must follow that every infinity is, in a way we cannot express, made finite to God.”

-Augustine, De Civitate Dei

God created the universe. The Torah says, “In the beginning God created the heaven and the earth” (1 Kings 7:23). Everything that we know, that we can understand, was made by God. The light, the dark, the plants and animals. We were all created by God. While Judaism teaches that God is everywhere, that He is in everything that we see and do, we also know that He is much more than what we can sense. He must exist beyond His own creation, and so He must exist beyond everything that we know. Even within the universe, we still know only a small part, and whatever may be beyond our comprehension of the physical world is a mystery to us. This boundary between the universe and the beyond or, better, the known and the unknown is a continuously shifting line. As scientists discover new properties of the world and as mathematicians discover new theorems and concepts, the known world expands into the unknown. Imagine that the universe, or what we understand of it, is a set within a set. Imagine that it is an expanding system within another system, a set within a set. Our knowledge of the universe is constantly and continuously expanding, so the magnitude of our understanding is growing. Yet, as we learn new things, we only introduce further unknowns in what is an
unending cycle. Augustine, in his quote, proposes that whatever is infinite in our understanding is made finite to God. Going further, an argument can be made that what is infinite is not somehow made finite to God, but that it is embodied by God or even that it is God.

Many scholars notice a parallelism between mathematics and Judaism. Hyman Gabai, in his 2002 book *Judaism, Mathematics, and the Hebrew Calendar*, compared not just the Jewish people and mathematicians to each other, but made a strong comparison between the larger systems themselves. “Judaism,” he said, “can be viewed as a deductive system with the fundamental undefined term of God, the Torah contents as axioms, and the Talmud as theorems derived from axioms” (Lesser). Mathematics and Judaism reveal different versions of what our reality is composed of by employing similar systems of study. While mathematics was built surrounding the number system, Judaism was built surrounding the word of God and the Torah. Rabbi Joseph B. Soloveitchik takes a similar position to Gabai in comparing what he calls his Jewish “halakhic man” to mathematicians

“Halakhic man, well furnished with rules, judgements, and fundamental principles, draws near the world with an a priori relation. His approach begins with an ideal creation and concludes with a real one. To whom may he be compared? To a mathematician who fashions an ideal world and then uses it for the purpose of establishing a relationship between it and the real world” (Soloveitchik, 19).

The two similarly functioning systems of thought cross paths at many points, such as in the Kabbalistic practice of gematria where letters of the Hebrew alphabet are assigned numerical values to find further meaning in the words of the Torah. Parallelism between the two complex institutions, mathematics and Judaism, appears many times within the world and our
understanding of it. Helplessly entangled, the relationship between mathematics and Judaism repeats and forms a fractal pattern. A fractal pattern is a pattern which possesses self-similarity. This means that the pattern repeats itself on every scale, so a small portion of the larger image will look the same as the entire image. Some naturally occurring fractal patterns occur in seashells, snowflakes, and lightning. When weighing-in on the issue of infinity, mathematics and Judaism dance around each other and interact in a pattern that reflects the characteristics of a fractal.

Let’s now assume that the universe and all that exists within it and beyond it is the complete image that is a fractal and has self-similarity. As discussed previously, the universe can be divided into parts that are known and understood by people and those that are not. As advancements are made in the sciences, we fill the gaps in our knowledge as human beings. At the same time, though, we continue to discover more that we cannot understand. This lack of understanding of natural phenomena is generally the basis for an understanding of God and godliness in many religions. Even in the sciences, this idea tends to crop up before being quickly extinguished by scientific peers. In his collection *On the Gods and Other Essays*, Robert G. Ingersoll stated, “No one infers a god from the simple, from the known, from what is understood, but from the complex, from the unknown, and incomprehensible. Our ignorance is God; what we know is science.” Each day, mankind is expanding our knowledge of different aspects of the universe, yet even as the ‘known’ is expanding, the ‘unknown’ will never be entirely diminished. What we know about the world is in fact a mere approximation of reality. Our entire understanding of the universe comes from approximations, and beyond the limits of that understanding is what many people call God.
We presume to know that God is infinite, and the Kabbalists are strong proponents of this idea. From Kabbalah we learn that, “the great entity that is God … is so large, so supreme, so far beyond description, that it is given the only name [they] could possibly use to describe it: Ein Sof. The two words mean *Infinity. God is infinite*” (Aczel, 34). Following the proposition that God is infinite, is the question of whether we can comprehend something that is infinite. Can we understand an infinite God? Maimonides contradicted himself in that, “on the one hand, [he] designated the knowledge of the Creator as the guiding criteria for man, as his ultimate end,” while, “on the other hand, [he] held the view that knowledge of God is not in the realm of human cognition” (Soloveitchik, 11). To understand an infinite being or an infinite anything, there need to be finite elements that can be understood. Mathematics has integers along with basic laws for how they can be interpreted. Infinity in its most basic conception is found by adding one continuously to a number with no end. Similarly, Judaism answers this question with the ten Sefirot. They “are the building blocks of creation, the archetypes of existence, the traits of God, and the primary values of the world.” (Drob). Despite the infinitude of God, an understanding of these ten sefirot can bring us closer to understanding Him. Though the traits cannot fully embody the grandness of God, they act as an approximation allowing for further comprehension. Some Kabbalists “speak of them as being one with Ein-sof, in the sense that the flame, the sparks, and the aura are one with the fire” (Drob). Using information or characteristics that we know and applying them to some unknown in order to better our understanding is a tool people use to grasp new ideas, whether they’re mathematical, Jewish or otherwise. Assigning ten concrete qualities that people can understand, and can possess, to an unknowable God can be compared to making an approximation. It is widely understood that God cannot, in truth, be limited to these ten aspects, but it is these ten aspects which point to the rest.
Mathematics is a complicated system, and just as with other systems it possesses a fundamental philosophical question, one that is relevant to our own arguments here. Is mathematics discovered or invented? The Platonic view of this question is that, “Just as electrons and planets exist independently of us, so do numbers and sets. And just as statements about electrons and planets are made true or false by the objects with which they are concerned and these objects' perfectly objective properties, so are statements about numbers and sets. Mathematical truths are therefore discovered, not invented” (Linnebo). The opposing viewpoint, non-Platonism, suggests that mathematics is a human construct that we then use to describe reality. For the sake of argument, let’s assume a Platonian view of mathematics as it is the view held by many mathematicians and scientists. “Mathematics has been called the language of the universe. Scientists and engineers often speak of the elegance of mathematics when describing physical reality, citing examples such as π, E=mc2, and even something as simple as using abstract integers to count real-world objects” (Zyga). Still, these formulas and numerical representations are many times not fully accurate. They are simply representative of an ideal world that the mathematician studies to better understand reality.

This idea is discussed with elegance in Rabbi Joseph Soloveitchik’s *Halakhic Man* in which he compares the Jewish “Halakhic Man” to mathematicians. “There exists an ideal world and a concrete one,” he says, “and between the two only an approximate parallelism prevails. In truth, not only from a theoretical, ideal perspective does mathematics pay no attention to concrete correlatives, but even from a utilitarian standpoint the mathematical approach has no desire to apprehend the concrete world per se but seeks only to establish a relationship of parallelism and analogy” (Soloveitchik, 19). Halakhic man, Soloveitchik says, is like a mathematician in the ways he approaches the world, the Torah, and God. He uses similar
phrasing in the way that he describes halakhic man’s approach to the world. “His approach to reality consists solely in establishing the correspondence in effect between his ideal, a priori creation and concrete reality” (Soloveitchik, 18). Both the mathematician’s approach to the world and halakhic man’s approach to reality are a form of approximation. Approximation across many fields points to a greater understanding of a concept. Dr. M. Jagadesh Kumar, in advocating for the teaching of the importance of approximations to science students, said, “approximations are the lifeline to solve the most complicated aspects of nature” (Kumar). As such, mathematicians and those following the Jewish traditions face similar questions and take a similar approach in answering them. In making approximations, in any field, some accuracy is lost. The gap between the approximation and reality is equivalent to a truncated remainder, and is repeatedly attributed to God regardless of the situation.

Our second embodiment of the fractal comes in the form of a mathematical exploration of infinity. The mathematical concept of infinity is complex and is still outside the realm of our understanding. The basic idea of infinity is simple, even small children can comprehend it. Children arguing with their parents over who loves the other more quickly arrive at the idea of infinity. One loves the other times a million, then times a gagillion, a gagillion and one, and finally, times infinity. We know conceptually, even from a young age, that you can add one to any number and it will produce a larger number and that you can do this forever. Infinity, though, is much more complicated. In the seventeenth century, Galileo introduced a new property of infinity. He found that the set of all whole numbers is ‘equal’ to the set of all squares of whole numbers by assigning a one-to-one correspondence between a number and its square. Just as Galileo understood that, “an infinite set can be in a sense ‘equal’ to a part of itself,” Amir Aczel proposes that, “perhaps this is something the Kabbalists had in mind when they said the
ten Sefirot are a part of the infinity of God. If God is infinite, certainly extracting ten elements still leaves an infinite set” (Aczel, 56). In these simple constructions and with this simple comprehension of infinity, we already see more intricate correlations of mathematical concepts to the study of Jewish traditions.

In examining the irrational numbers, numbers like pi and e, further properties of infinity are revealed. Georg Cantor, arguably one of the most influential mathematicians of the nineteenth century, discovered and proved many properties of infinity and simultaneously introduced countless more questions. Considered to be the father of Set Theory, Cantor used his ideas about sets to discover that there are different magnitudes of infinity. In 1874, Cantor published a paper in which, “the main result was that … the transcendental irrational numbers, could not be enumerated—their order of infinity was higher than that of the rational and algebraic numbers” (Aczel, 117). At the time, Cantor’s work was highly disputed, but now we see that what he was working on was the beginning of man’s discovery of actual infinity. Once Cantor found that there were different “levels” of infinity, he needed a symbol to define the cardinalities of each infinity. So, “[he] hypothesized the existence of a sequence of alephs. He named the lowest order of infinity, the infinity of the integers and the rational (as well as algebraic) numbers, aleph-zero. It is written $\aleph_0$” (Aczel, 146). He was then able to prove some basic properties of aleph-zero and laws of arithmetic. Still, he spent much of the rest of his life working to prove further order among his transfinite numbers. Much of the contention for Cantor’s work came from mathematicians who were made uneasy by the concept of infinity, and Cantor’s own time spent on the problem led to multiple stays in a mental institution throughout his later years. One of his final discoveries, after a lifetime of additions to the field of mathematics, was that there can be no largest set (each set leads to a larger set of subsets), from
which one can infer that there can be no greatest $\aleph$. Cantor concluded close to the end of his life, in his own words, that, “I have never proceeded from any ‘Genus supremum’ of the actual infinite. Quite the contrary, I have rigorously proved that there is absolutely no ‘Genus supremum’ of the actual infinite. What surpasses all that is finite and transfinite is no ‘Genus’; it is the single, completely individual unity in which everything is included, which includes the Absolute, incomprehensible to the human understanding. This is the Actus Purissimus, which by many is called God” (Aczel. 189). According to Cantor, beyond the numbers are countable levels of infinity and beyond that there is only God. Here is where the repeating pattern of the incomprehensible, unknown phenomena in the world being attributed to God reveals itself again.

Finally, an even closer view of our fractal comes from looking at just one number. Within the number system, there is a particular class of numbers called irrational numbers which have a certain infinite quality. An irrational number is one which can’t be written as the ratio of two integers. In decimal form, an irrational number has “an infinite number of digits to the right of the decimal point, without repetition” ("Irrational Number."). Pi, one of the most famous irrational numbers, is unique as it also appears in the Torah. More than just being mentioned in the Torah, the notorious irrational was approximated which resulted in further examination and analysis by many scholars and rabbis. A value for pi is never explicitly stated, but the irrational number is referenced in I Kings 7:23 where a pool in King Solomon’s Temple is being described:

“And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it round about” (I Kings 7:23).
With a circumference of thirty cubits and a diameter of ten cubits, the ratio between the two (which we would expect to be the irrational pi, 3.141592...) is the integer 3. Thanks to rigorous mathematical proofs we know not only that 3 is only an approximation for the value of pi but that it is not a very good approximation, as it is most commonly recognized to be close to 3.14.

Stemming from this one passage in the Torah is list of many intensive explanations for what the approximation in the passage, of what we know to be pi, means. One theory that was shared is the notion that it was Phoenician artisans that built the pool and so the author of the passage wouldn’t know the exact dimensions. Another author offered an alternate formula to determine the diameter and a “totally unwarranted assumption that the cauldron was perfectly cylindrical” which essentially just transferred the approximation from pi to the diameter which comes out as around 9.5 cubits rather than 10 as stated (Dutch). A different approach was taken by Imry Galeinai who didn’t seek to “explain away” the approximation. He sought, instead, to “tell the story of [pi] from the perspective of its remainder” (Galeinai). He proposed the idea that to understand the significance of the passage, attention should be focused on what the approximation points to. As he defines it, “[the remainder] represents the part of pi that runs on forever and ever after the decimal point.” To understand what he means, one must visualize a hexagon inscribed in a circle with radius 1. Then, the hexagon’s circumference is 6 and width is 2. The ratio between these two numbers gives 3, the approximate value of pi referenced in the Torah. The ratio between the circle’s circumference and its diameter is, of course, pi. In Galeinai’s analogy, we can understand the significance of this by understanding the six sides of the hexagon as correlating to the six days of the week and the six days of creation. Then, “the remainder of [pi], reflected by the difference between the circumference of the circle and the circumference of the
hexagon, represents the Shabbat” (Galeinai). While a circle may appear to have one curved side connected to itself, in mathematics it is a polygon with infinitely many sides. Galeinai attributes the one side of the circle to Shabbat and the holiness of the day. Another conclusion one can draw is that the infinite sides of the circle are representative of the infinitude of God Himself.

God is in everything that we know, but he is also more than that. Beyond everything that we can understand, and everything that we can ever dream to understand, is God. Mathematics and Judaism, while hopelessly connected, cannot be compared with a one-to-one correspondence, though Gabai and Soloveitchik both come close, comparing the larger systems and comparing how mathematicians and halakhic Jews may navigate within the systems. The similarities between them lead to similar fundamental questions, such as whether the institution is discovered by man or invented by man or whether it’s possible to comprehend the infinite. Still, there may not be a pattern that can equate the two with precision. In examining the fractal that connects the understanding of infinity in math and Judaism, this lack of precision is apparent. The complete image is the easiest to see and understand because it encompasses everything in existence. The image is of the universe and man’s unlimited expansion of knowledge into an even greater infinitude, which is God. Within the number system, the same pattern repeats. The integers, we know, expand on to infinity. Beyond the concrete integers are the cardinalities of levels of infinity, and beyond that knowledge is the Absolute, as Cantor called it, or God. On an even smaller scale, the irrational numbers also exhibit a similar pattern. The decimals of pi go on without end and without repetition, but is often approximated. In analysis of the irrational number’s appearance in the Torah, the difference between what we can comprehend (the approximation) and what we can’t (the actual value), the truncated remainder, is attributed to God. Within these three scales of the pattern, though, there are many other
equivalences between Judaism and mathematics which add to the argument. Ultimately, the fractal pattern shows that God is infinite and man will never know all that God is. Even as we expand our understanding of the universe, our understanding of God will never be more than an approximation, comparing our reality with our ideal of Him.
Works Cited


