The deadline for submitting the solutions is Dec 14, 12pm. The solutions are to be submitted electronically (scanned hand-written solutions are fine). E-mail i.loseu@neu.edu.

There are three problems with total number of points equal to 30. The maximal number of points you get for this problem set is 20. 50% of your score above 20 will count to augment your scores from previous psets. Partial credit is given.

**Problem 1.** Prove the Krull-Schmidt theorem (Theorem 1.1 from Lecture 16) (5pts).

**Problem 2.** Suppose a quiver $Q$ contains no oriented cycles. Then the dimension of an irreducible representation is a simple root and the number of isomorphism classes of irreducible representations coincides with the number of vertices (3pts).

**Problem 3.** Prove the Kac theorem using linear algebra for the following quivers:
1) Type $A_m$ quiver oriented left to right (2pts).
2,3) Quivers a),d) from Pic. 1.2 to Lecture 16, 2 pts each.
4) The Kronecker quiver with two vertices and two arrows in the same direction. This case is harder.

**Problem 4.** Let $G$ be a connected algebraic group acting linearly on a vector space $V$ with finitely many orbits. Show that $G$ acts on $V^*$ with finitely many orbits and that the number of orbits in $V^*$ coincides with the number of orbits in $V$ (4pts).

**Problem 5.** Check if there is a solution to the DS problem in the following cases (2pts per case). In which cases the number of solutions $Y_1, \ldots, Y_k$ is finite (2pts)? Below we indicate the dimension of the space and representatives of conjugacy classes.

a) $n = 2; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

b) $n = 4, \text{diag}(1, 1, 0, 0), \text{diag}(-1, -1, 0, 0), \text{diag}(2, 2, 0, 0), \text{diag}(-2, -2, 0, 0)$.

c) $n = 4, \text{diag}(2a, 0, 0, 0), \text{diag}(b, b, 0, 0), \text{diag}(c, c, 0, 0), \text{diag}(d, d, e, e)$, where $a, b, c, d, e$ are generic with $a + b + c + d + e = 0$.

d) $n = 4, C_1 = O_{(2,1,1)}, C_2 = C_3 = C_4 = O_{(2,2)}$ (where the notation $O_{\lambda}$ means the nilpotent orbit corresponding to the diagram $\lambda = (\lambda_1, \ldots, \lambda_k)$).