Traces of intertwiners for quantum affine $\mathfrak{sl}_2$ and Felder-Varchenko functions

Abstract. This talk concerns two approaches for studying a family of special functions occurring in the study of the q-Knizhnik-Zamolodchikov-Bernard (q-KZB) equation. The philosophy of KZ-type equations predicts that it admits solutions via (1) traces of intertwining operators between representations of quantum affine algebras produced by Etingof-Schiffmann-Varchenko and (2) certain theta hypergeometric integrals we term Felder-Varchenko functions. In a series of papers in the early 2000’s, Etingof-Varchenko conjectured that these families of solutions are related by a simple renormalization; in the trigonometric limit, they proved such a link and used it to study these functions.

In recent work, I resolve the first case of the Etingof-Varchenko conjecture by showing that the traces of quantum affine $\mathfrak{sl}_2$-intertwiners of Etingof-Schiffmann-Varchenko valued in the 3-dimensional evaluation representation converge in a certain region of parameters and give a representation-theoretic construction of Felder-Varchenko functions. I will explain the two constructions of solutions, the methods used to relate them, and connections to affine Macdonald theory and the Felder-Varchenko conjecture on the q-KZB heat operator and corresponding $\text{SL}(3, \mathbb{Z})$-action.

This talk is based on the preprint arXiv:1508.03918.