The Kodaira dimension of complex hyperbolic manifolds

Abstract. Complex hyperbolic manifolds are manifolds of constant negative curvature; they are uniformized by the complex ball. In dimension one, sufficiently punctured rational and elliptic curves are hyperbolic, and a famous example of Hirzebruch shows that there are also infinitely many hyperbolic surfaces with Kodaira dimension 0. In joint work with J. Tsimerman, we show that in dimension $n \geq 3$ every hyperbolic manifold is of general type. Such a manifold $X$ admits a unique toroidal compactification $\overline{X}$ with boundary divisor $D$, and we further show that $K_{\overline{X}} + (1-t)D$ is ample for $0 < t < \frac{n+1}{2n}$—in particular $K_{\overline{X}}$ is ample for $n \geq 6$. The proof uses a hybrid technique employing both the hyperbolic geometry of the uniformizing group and the algebraic geometry of the toroidal compactification. We will also discuss applications to bounding the number of cusps and the Green–Griffiths conjecture.