NORTHEASTERN UNIVERSITY
DEPARTMENT OF MATHEMATICS

Prof. A. Suciu **MTH 3105 — TOPOLOGY 1** **Fall 1998**

FINAL EXAM

This is a take-home exam, due Wednesday, December 16. Do 6 out of the following 7 problems. Give complete proofs or justifications for each statement you make. Show all your work.

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1. Let K and L be finite simplicial complexes. Show that:
$$\chi(|K| \times |L|) = \chi(|K|) \cdot \chi(|L|).$$

2. Let $T_g^r = \#_1^g \mathbb{T}^2 \setminus \bigcup_1^r D^2$ be the orientable surface of genus g with r holes in it.
   (a) Find the fundamental group of T_g^r.
   (b) Find the homology groups of T_g^r.

3. Let $X = \mathbb{T}^2/\{p, q, r\}$ be the topological space obtained from the torus by identifying 3 distinct points on it.
   (a) Find the fundamental group of X.
   (b) Find the homology groups of X.

4. Consider the following labeled polygons, with edges identified in pairs, as follows:
   (i) $abcde^{-1}f^{-1}gha^{-1}b^{-1}c^{-1}d^{-1}efg^{-1}h^{-1}$.
   (ii) $abcde^{-1}f^{-1}gha^{-1}b^{-1}c^{-1}d^{-1}efgh$.
   For each of the resulting surfaces, determine its orientability status (i.e., orientable or not), its genus (i.e., number of handles or crosscaps), and its Euler characteristic.

5. Let $p : \widetilde{X} \to X$ be a covering map, with $p^{-1}(x)$ finite, for all $x \in X$. Show that:
   (a) $\widetilde{X}$ is Hausdorff if and only if X is.
   (b) $\widetilde{X}$ is compact if and only if X is.

6. Let K be the Klein bottle.
   (a) Find two non-equivalent 4-fold covers K, $p_1 : K_1 \to K$ and $p_2 : K_2 \to K$.
   (b) For each of those covers, find the subgroup $p_{i*}(\pi_1(K_i))$ of $\pi_1(K)$.
   (c) For each of those covers, determine the orientability status, the genus, and the Euler characteristic of K_i.

7. Let $T_g = \#_1^g \mathbb{T}^2$ be the closed, orientable surface of genus g. Show that:
   (a) There is a fixed-point-free action of $\mathbb{Z}_2$ on T_3.
   (b) The orbit space, $T_3/\mathbb{Z}_2$, is homeomorphic to T_2.
   (c) Show that $p : T_3 \to T_2$ is a covering projection.
   (d) (**Bonus**) Compute $p_* : \pi_1(T_3) \to \pi_1(T_2)$.