Prof. A. Suciu **Name:** ____________________

MTH 1733 # QUIZ 4 **Fall 1997**

1. $\boxed{\text{6 points}}$ Solve the following initial value problem:

$$y' - 3y = 3x + 5, \quad y(0) = 4.$$

2. $\boxed{\text{6 points}}$ Find the general solution of the following system of differential equations:

$$\begin{aligned} x' &= 2x + y \\ y' &= -x + 4y \end{aligned}$$

3. $\boxed{\text{6 points}}$ A tank initially contains 20 kg of salt dissolved in 200 liters of water. A brine solution containing 3 kg of salt/liter flows into the tank at the rate of 2 liters/minute. The mixture, kept uniform by stirring, flows out at the rate of 4 liters/minute. Set up an appropriate one-compartment model and write down the corresponding initial value problem (do not solve).

4. $\boxed{\text{6 points}}$ A certain bacteria culture grows at a rate proportional to its size, doubling every hour. The culture contains 5 million bacteria at time $t = 0$ (with time in hours).

   (a) Write down an initial value problem that models the growth of the bacteria culture.

   (b) Solve the intial value problem from part (a).

   (c) At what time will there be 200 million bacteria present?

5. $\boxed{\text{6 points}}$ Consider the following autonomous differential equation:

$$y' = y^2 - 2y - 3$$

(a) Find the equilibrium solutions, specifying whether they are stable or unstable.

(b) Sketch the solution curves corresponding to the initial values $y(0) = -2$, $y(0) = 2$, $y(0) = 4$.

(c) For each of the three curves $y = y(t)$ in part (b), decide whether an inflection point occurs, and, if it does, at what value of y.