Prof. A. Suciu

MTH 1230 **LINEAR ALGEBRA** **Spring 1999**

QUIZ 5

1. [14 points] Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$.

(a) Find a basis for $\ker A$.

(b) Find a basis for $(\ker A)^{\perp}$.

(c) Find a basis for $\ker A^{\top}$.

(d) Find a basis for $(\ker A^{\top})^{\perp}$.

(e) Which one of the above four linear subspaces—$\ker A$, $(\ker A)^{\perp}$, $\ker A^{\top}$, $(\ker A^{\top})^{\perp}$—equals $\operatorname{im} A$, and which one equals $\operatorname{im} A^{\top}$?

(f) What is the area of the parallelogram spanned by the column vectors of $5I_2 - A$?

2. [10 points] A company gathers the following data:

Year	1995	1996	1997	1998
Annual Sales (in millions of dollars)	2.0	2.5	3.2	4.1

Represent the years $1995, 1996, 1997, 1998$ as $0, 1, 2, 3$, respectively, and let x denote the year. Let y denote the annual sales (in millions of dollars).

(a) Find the least squares line relating x and y.

(b) Use the equation obtained in part (a) to estimate the annual sales for the year 2000.

3. [6 points] Let A and B be two 3×3 matrices, with $\det A = -2$ and $\det B = 0$.

(a) Is A invertible? If yes, compute $\det(A^{-1})$. If not, say so.

(b) Is B invertible? If yes, compute $\det(B^{-1})$. If not, say so.

(c) Compute: $\det(4A) =$

(d) Compute: $\det(A^4) =$