Prof. A. Suciu

MTH 1230 | **LINEAR ALGEBRA** | **Spring 1999**

QUIZ 4

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1. [10 points] Let A be an orthogonal $n \times n$ matrix (recall this means that the columns of A are orthonormal), and let $A^\top$ be its transpose.

(a) Find:

$AA^\top =$

$A^\top A =$

(b) Find:

$\dim(\ker A) =$

$\dim(\operatorname{im} A) =$

(c) Is $A^\top$ also orthogonal? Explain your answer.

(d) Are the rows of A orthonormal? Explain your answer.

(e) The QR-factorization of A is given by:

$Q =$

$R =$

2. 6 points Find all orthogonal matrices of the form

$$A = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & a \\ -\frac{\sqrt{2}}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & b \\ \frac{\sqrt{2}}{2\sqrt{3}} & -\frac{1}{\sqrt{3}} & c \end{bmatrix}$$

3. 14 pts Let $\vec{v}_1 = \begin{bmatrix} 5 \\ 12 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}.$

(a) Find the lengths of $\vec{v}_1$ and $\vec{v}_2$, and compute the dot product $\vec{v}_1 \cdot \vec{v}_2$.

(b) Find unit vectors in the direction of $\vec{v}_1$ and $\vec{v}_2$, respectively.

(c) Find the angle between $\vec{v}_1$ and $\vec{v}_2$.

(d) Find the projection of $\vec{v}_2$ onto the subspace of $\mathbb{R}^2$ spanned by $\vec{v}_1$.

(e) Let $A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix}$. Use the Gram-Schmidt process to find the QR-factorization of A.