Prof. A. Suciu

MTH 1230 **LINEAR ALGEBRA** **Spring 1999**

QUIZ 3

1. 11 points Let $A = \begin{bmatrix} 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$.

(a) Find a basis for $\operatorname{im} A$.

(b) Find a basis for $\ker A$.

(c) Compute:

$\dim(\operatorname{im} A) =$

$\dim(\ker A) =$

$\operatorname{rank} A =$

2. [10 points] Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -5 & -1 \\ -1 & 4 & 5 \end{bmatrix}$.

(a) Determine whether the column vectors of A are dependent or independent. If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

(b) Does the equation $A \cdot \vec{x} = \vec{0}$ only have the solution $\vec{x} = \vec{0}$, or does it have other solutions? Explain your answer.

(c) Does the equation $A \cdot \vec{x} = \vec{b}$ have a solution for every choice of $\vec{b}$ in $\mathbb{R}^3$? Explain your answer.

3. [9 points] In each of the following, a subset V of $\mathbb{R}^2$ is given. Circle one answer:

(a) $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; x - 2y = 6 \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^2$: YES NO

(b) $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; \begin{array}{c} x - 2y = 0 \\ x, y \text{ integers} \end{array} \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^2$: YES NO

(c) $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; xy \geq 0 \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^2$: YES NO

(d) $V = \left\{ \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix} \;\middle|\; \begin{array}{c} x, y \text{ arbitrary} \\ \text{constants} \end{array} \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^2$: YES NO