Prof. Alexandru Suciu

MTH 1230 **LINEAR ALGEBRA** **Spring 2001**

EXAM 3

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1. 10 pts Consider the independent vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

Find an orthonormal basis $\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ for the subspace of $\mathbb{R}^4$ which has $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ as a basis.

2. 12 points Let $A = \begin{bmatrix} -3 & 4 \\ 9 & -12 \end{bmatrix}$.

(a) Find a basis for $\ker A$.

(b) Find a basis for $(\ker A)^{\perp}$.

(c) Find a basis for $\ker A^{\top}$.

(d) Find a basis for $(\ker A^{\top})^{\perp}$.

3. 8 points Find all pairs of orthonormal vectors of the form $\vec{v}_1 = \begin{bmatrix} a \\ a \\ a \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ b \\ c \end{bmatrix}$

[**Warning**: The numbers a, b, c can very well be negative!]

4. 6 points Which of the following statements are true, for **all** $n \times n$ orthogonal matrices A?

(a) $\operatorname{rref} A = I_n$

(b) $\ker A = \{\vec{0}\}$

(c) $\operatorname{im} A = \{\vec{0}\}$

(d) $A \cdot A^\top = A^\top \cdot A$

(e) $A^{-1} = A$

(f) $(A^\top)^{-1} = A$

5. 14 pts Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix}$.

(a) Find the least squares solution $\vec{x}^*$ of the inconsistent system $A \cdot \vec{x} = \vec{b}$.

(b) Find the 3×3 matrix associated with the projection of $\mathbb{R}^3$ onto the subspace $\operatorname{im} A$.

(c) Find the projection of the vector $\vec{b}$ onto $\operatorname{im} A$.