PROF. A. SUCIU MTH 1230 – LINEAR ALGEBRA FALL 1997

SAMPLE QUIZ 4

1. Find the least squares solution of the inconsistent system $A\mathbf{x} = \mathbf{b}$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$

2. Find the equation of the least-squares line that fits the following data points:

x	1	2	4	5
y	0	1	2	3

Sketch the resulting line. What is the predicted value of y at $x = 6$, based on this model?

3. Let $\quad \mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$

   (a) Find an orthonormal basis for the subspace of $\mathbb{R}^4$ which has $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ as a basis.

   (b) Use part (a) to find the QR factorization of the matrix $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$.

4. Let

$$A = \begin{bmatrix} 4 & -7 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

   (a) Find the eigenvalues of A.

   (b) Find a basis for each eigenspace of A.

   (c) Find a diagonal matrix Λ and an invertible matrix S such that $A = S \cdot \Lambda \cdot S^{-1}$.
   [You do not have to calculate S^{-1}.]

5. A 2×2 matrix A has first row $\begin{bmatrix} -2 & 5 \end{bmatrix}$ and eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$.

   (a) Find A.

   (b) What are the eigenvalues of A^{-1}?

   (c) Compute $\det(A^{-1} + I)$, where I is the identity 2×2 matrix. Explain your result.

6. A 4×4 matrix has eigenvalues $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3, \lambda_4 = 4$.

   (a) Find the eigenvalues of A^2.

   (b) Find the trace of A^2.

   (c) Find the determinant of A^2.