QUIZ 4

1. $\boxed{\text{8 points}}$ Let $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$.

(a) Find the least squares solution $\left[\begin{smallmatrix} \bar{x} \\ \bar{y} \end{smallmatrix}\right]$ of the inconsistent system $A \cdot \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] = \mathbf{b}$.

(b) Use your answer to part (a) to find the projection of $\mathbf{b}$ onto $\mathrm{CS}(A)$, the column space of A.

2. $\boxed{\text{8 points}}$ A company gathers the following data:

Year	1993	1994	1995	1996	1997
Annual Sales (in millions of dollars)	0.8	2	3	4.2	5

Represent the years $1993, \ldots, 1997$ as $-2, -1, 0, 1, 2$, respectively, and let x denote the year. Let y denote the annual sales (in millions of dollars).

(a) Find the least squares line relating x and y.

(b) Use the equation obtained in part (a) to estimate the annual sales for the year 1999.

3. $\boxed{\text{8 points}}$ Apply the Gram-Schmidt process to the vectors $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and write the result in the form $A = Q \cdot R$.

4. $\boxed{\text{8 points}}$ Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) Find a basis for each eigenspace of A.

(d) Find a diagonal matrix Λ and an invertible matrix S such that $A = S \cdot \Lambda \cdot S^{-1}$.
[You do not have to calculate S^{-1}.]

5. $\boxed{\text{8 points}}$ Let A be a 3×3 matrix, with eigenvalues $\lambda_1 = -2$, $\lambda_2 = 0$, $\lambda_3 = 5$.

(a) Compute $\operatorname{tr}(A)$ and $\det(A)$.

(b) Is A invertible? Explain your answer.

(c) Is A diagonalizable? Explain your answer.

(d) Compute $\operatorname{tr}(A^3)$ and $\det(A^3)$.