Prof. A. Suciu | **MTH 1187–Probability** | **Winter 2001**

Practice Problems for Exam 3

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(1) A random variable has probability distribution given by

x	-1	0	1	2	3
$P(X=x)$	$1/8$	$1/4$	$1/4$	$1/4$	$1/8$

Find the mean and standard deviation of X.

(2) Random variables X and Y have joint pdf

$Y\backslash X$	1	3
-2	$1/5$	$1/10$
2	$2/5$	$3/10$

Find: $E(X)$, $E(X^2)$, $V(X)$, $E(Y)$, $E(Y^2)$, $V(Y)$, $E(XY)$. Are X and Y independent?

(3) A random variable X has $E(X)=5$ and $E(X^2)=40$. Let $Y=-6X+10$. Compute $E(Y)$ and $V(Y)$.

(4) Let X and Y be independent random variables, with: $E(X)=2$, $E(Y)=6$, $V(X)=9$, $V(Y)=16$. Compute:

(a) $E(3X-4Y-5)=$

(b) $V(3X-4Y-5)=$

(c) $E(X^2-Y^2)=$

(d) $D\left(\dfrac{X+Y}{2}\right)=$

(5) A random variable X can have values $-3,-2,-1,1,2,3$, each with probability $1/5$. Let $Y=X^2-3$. Find the density, the mean, and the variance of Y.

(6) Random variables X and Y have means -3 and 8, and standard deviations 1 and 5.

(a) Find $E(X^2)$ and $E(Y^2)$.

(b) Let $Z=4X-6Y+7$. Find $E(Z)$ and $V(Z)$.

(7) A pointer is spun on a fair wheel of chance numbered from 0 to 100 around its circumference.

(a) What is the average value of all possible pointer positions?

(b) What deviation from its average value will pointer position take on the average?

(8) Suppose that a word is to be picked randomly from the following sentence:

> Probability theory began in seventeenth century France when two great French mathematicians, Blaise Pascal and Pierre de Fermat, corresponded over two problems from games of chance.

(a) What is the expected value of the length of the word picked?

(b) What is the standard deviation of the length of the word picked?

(9) A little deck has 6 cards: 2 Aces, 2 Kings, and 2 Queens. Two cards are drawn at random, without replacement. If X is the number of Aces obtained, find $E(X)$ and $V(X)$.

(10) The voltage in a certain circuit is a random variable with mean 120 and standard deviation 5. Sensitive equipment will be damaged if the voltage is not between 112 and 128. Use Chebyshev's inequality to bound the probability of damage.

(11) In a certain casino game, you win \$2 with probability 0.3 and lose \$1 with probability 0.7. You play 100 times (independently). Find the mean and standard deviation of your total net winnings.

(12) An insurance company sells life insurance to 500 customers. They charge each customer \$300. If the customer dies this year, the company pays out \$10000. Suppose the probability that any individual customer dies this year is 1%, and that all customers live and die independently. Let Q be the company's profit. Find the expected value and standard deviation of Q.

(13) A random number generator is used to generate 360 random numbers from the interval $[0, 1]$. Use Chebyshev's inequality to find a lower bound for the probability that the sum of the numbers lies between 170 and 190.

(14) A binary transmissiom channel introduces errors with probability 0.1. Use Chebyshev's inequality to estimate the probability that there are between 4 and 16 errors in 100 bit transmissions.

(15) A fair die is rolled 800 times. An outcome of 0 or 1 is considered a success; other outcomes are failures. Let X be the number of successes.

(a) Find $E(X/800)$ and $VAR(X/800)$.

(b) Use Chebyshev's inequality to bound the probability that the proportion of successes is between 17/60 and 23/60.

(16) A biased coin with $P(\mathrm{H}) = 0.4$ is tossed 100 times. Let X be the number of heads in the 100 tossings.

(a) Use Chebyshev's inequality to find an upper bound for $P(X \leq 30 \text{ or } X \geq 50)$.

(b) Use Gaussian approximation to compute $P(X \leq 30 \text{ or } X \geq 50)$.

(17) Let X_1 be a random number between 0 and 1. It turns out that $E(X_1^2) = 1/3$.

(a) Find $E(X_1)$ and $V(X_1)$.

(b) If $X_1, \ldots, X_{400}$ are independent, each with the same pdf as X_1, and $\overline{X} = \frac{1}{400}(X_1 + \cdots + X_{400})$, find $E(\overline{X})$ and $V(\overline{X})$.

(18) Flip a fair coin 3000 times. What does Chebyshev say about the probability that your fraction of heads will be between 0.46 and 0.54? How about the Central Limit Theorem?

(19) A fair coin is tossed 1000 times. Use the Central Limit Theorem to approximate the probability that bewteen 480 and 520 heads are obtained. How does this compare to Chebyshev's bound?

(20) About 2% of a certain type of RAM chips are defective. A student needs 50 chips for a certain board. How many should she buy in order for there to be a 99% chance or greater of having at least 49 working chips?