

Practice Problems for Exam 2

- (1) Let A and B be events such that $P(A) = 0.7$, $P(B) = 0.4$, $P(A \cup B) = 0.9$.
- Find $P(A \cap B)$, $P(A | B)$, $P(B | A)$.
 - Are A and B mutually exclusive? Why, or why not?
 - Are A and B independent? Why, or why not?
- (2) Two real numbers, X and Y , are picked at random between 0 and 5. Find:
- $P(\max(X, Y) < 1)$
 - $P(\min(X, Y) > 2)$
 - $P(X + 2Y < 4)$
 - $P(X + 2Y > 6)$
- (3) The lifetime of a component, in years, has pdf $f(t) = \frac{2}{(1+t)^3}$ and cdf $F(t) = \frac{t(t+2)}{(1+t)^2}$, for $t \geq 0$.
- What is the probability that the component lives at least 3 years?
 - What is the probability that a component that has lived 2 years will die before the age of 3 years?
- (4) Find the probability of drawing 3 aces from a deck of 52 cards if the cards are
- Replaced.
 - Not replaced.
- (5) In Megabucks, your chance of winning is about $1/1,900,000$. Suppose you and 1,000,000 other people play independently. What is the chance that you win **and** nobody else wins?
- (6) A biased coin which comes up heads three times as often as tails is tossed. If it shows heads, a chip is drawn from urn I which contains 2 white chips and 5 red chips. If the coin comes up tails, a chip is drawn from urn II which contains 7 white and 4 red chips. Given that a red chip was drawn, what is the probability that the coin came up heads?
- (7) A transmitter send binary bits, 80% 0's and 20% 1's. When a 0 is sent, the receiver will detect it correctly 80% of the time. When a 1 is sent, the receiver will detect it correctly 90% of the time.
- What is the probability that a 1 is sent and a 1 is received?
 - If a 1 is received, what is the probability that a 1 was sent?
- (8) At a large university in the Boston area, 30% of the students are engineers. If we select 10 students at random, what is the probability that
- Exactly 3 of them are engineers?
 - At least one of them is an engineer?
- (9) At yet another large urban University, the average SAT score of an entering freshman is 1,000, with a standard deviation of 100.
- What is the 95% percentile of the freshman class?
 - What percentage of those in the 1,100 club have an SAT score over 1,200?
- (10) An urn contains 10 black balls and 20 red balls. If we draw 3 balls from the urn, what is the probability that all 3 are red,
- If we replace each ball before drawing again?
 - If we do not replace?

- (11) In a certain city, 80% of all defendants are actually guilty. Furthermore, 90% of all guilty defendants are convicted, whereas 30% of all innocent defendants are convicted.
- What fraction of all verdicts are correct?
 - If you are convicted, what is the probability that you are guilty?
- (12) A consulting firm rents cars from two rental agencies: 60% from agency A and the rest from agency B. It so happens that 9% of the cars from agency A need a tune-up and 15% of the cars from agency B need a tune-up.
- What is the probability that the next car the firm rents will need a tune-up?
 - What is the probability that a car the firm rents is from agency B if we know that it needs a tune-up?
- (13) A box has four dice in it. Three of the are fair dice but the fourth has the number five on all of its faces. A die is chosen at random from the box and is rolled three times. Every time it is rolled a five turns up. What is the probability that this is the rigged die?
- (14) The result on a certain blood test is normally distributed with mean $\mu = 40$ and standard deviation $\sigma = 5$. What fraction of results are greater than 37?
- (15) English and American spellings are *rigour* and *rigor*, respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel. If 30% of the English-speaking men at the hotel are English and 70% are Americans, what is the probability that the writer is an Englishman?
- (16) Twenty people come to a party (independently). In each individual case, there is a 10% chance that the person will bring a present. What is the probability of **more** than 2 people bringing presents?
- (17) The voltage supplied to a piece of equipment is a Gaussian random variable with mean 120 and standard deviation 5. The equipment will be damaged if the voltage is outside the range [113, 123]. What is the probability of damage?
- (18) Every winter a man splits chunks of wood for a stove in his house. Over the years he has settled into a style of axe-splitting that split a chunk of wood with a probability of 0.6 per swing. What is the probability that, during his next 13 axe swings, he will get at most 3 splits?
- (19) A professor teaches Probability each year. For tests, he uses exams that he knows from past experience produce a grade distribution with mean 75 and standard deviation 7.
- What is the probability of a student getting a C grade, if the grade distribution is normal?
 - Where should the cutoff be between the A's and the B's, if the grade distribution is normal, and 20% of the class is to receive an A?
- (20) A computer company purchases $\frac{2}{3}$ of the chips that it uses from supplier A, and the remaining chips from supplier B. Chips supplied by A have a lifetime which is a Gaussian random variable with mean 8,000 hours and standard deviation 3,000 hours. Those supplied by B have a lifetime given by a Gaussian random variable with mean 10,000 hours and standard deviation 2,000 hours.
- Given that a chip is from A, what is the probability that its lifetime will be at least 9,000 hours?
 - Given that a chip is from B, what is the probability that its lifetime will be at least 9,000 hours?
 - What is the probability that the lifetime of a randomly selected chip will be at least 9,000 hours?
 - Given that the chip has lasted 9,000 hours, what is the probability that it came from A?