Name: ________________________ **Instructor:** ________________________

Department of Mathematics
Northeastern University

MTH 1187–Probability
Winter 2001

Final Exam

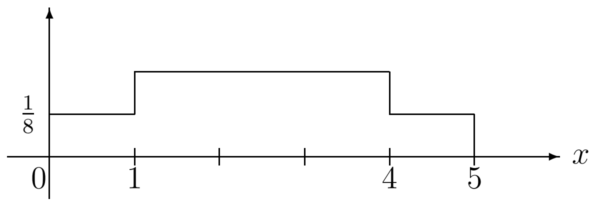
Instructions: This is an open-book, open-notes exam. Put your name, and the name of your instructor, in the blanks above. There are 5 problems, worth 40 points in all. **Show your work!** If there is not enough room, use the back page.

$\boxed{\text{8 pts}}$ **(1)** Jane and Jill each take 3 shots at a basket. Jane's success probability is $\frac{3}{5}$ on each shot, and Jill's is $\frac{1}{2}$. All shots are independent. Find:

(a) The probability that <u>at least one</u> of the 6 shots is a success.

(b) The expected total number of successes.

7 pts **(2)** Here is the density (pdf) of a random variable X.

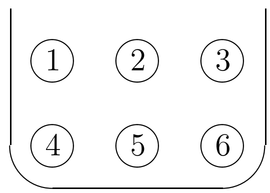


(a) Find $P(X < 3)$.

(b) Find the variance of X. [You can use $E(X^2) = \frac{47}{6}$.]

7 pts

(3) Two balls are picked at random from this box without replacing.



Suppose the numbers on the picked balls are X and Y. Find:

(a) $P(X+Y=6)$.

(b) $E(X+Y)$.

9 pts

(4) A coin is tossed 9 times.

(a) If the coin is fair, find the probability that 6 heads are obtained.

(b) If the coin is lopsided, so that the probability of heads is $\frac{2}{3}$ on any toss, find the probability of obtaining 6 heads.

(c) Before tossing, we believed that there was a 50% chance that the coin was fair, and a 50% chance that the coin was lopsided (with probability of heads $\frac{2}{3}$). Given that 6 heads were obtained, now what's the probability that the coin is fair?

$\boxed{\text{9 pts}}$ **(5)** In a certain game, you win \$2 with probability $\frac{1}{4}$, and lose \$1 with probability $\frac{3}{4}$. You play 100 times (independently). Let W be the <u>net total</u> winnings.

(a) Find the mean and the standard deviation of W.

(b) Use the Central Limit Theorem to approximate the probability that you come out ahead after 100 games.