

Instructions: This is an open-book, open-notes exam. There are 4 problems, each worth 10 points. Put your name in the blanks above. **Show your work!** if there is not enough room, use the back page. Give all numerical answers as **fractions**, or as decimals, correct to at least **3 significant digits**.

(1) A random variable X has $E(X) = -4$ and $E(X^2) = 30$. Let $Y = -3X + 7$. Compute:

(a) $V(X) =$

(b) $V(Y) =$

(c) $E((X + 5)^2) =$

(d) $E(Y^2) =$

(2) A deck has only face cards: 4 Kings, 4 Queens, and 4 Jacks. Two cards are drawn at random, without replacement. If Q is the number of Queens obtained, find the expected value, the variance, and the standard deviation of Q .

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- (3) In a certain casino game, you can win either \$5, with probability 0.05, or \$2, with probability 0.2, or lose \$1, with probability 0.75.
- (a) Find the mean and variance of your net winnings if you play once.
- (b) Suppose you play 80 times this game. Find the mean and standard deviation of your total net winnings.
- (c) Use Gaussian approximation to the probability you come out ahead after playing 80 times.
- (4) A biased coin comes up heads 30% of the time. The coin is tossed 400 times. Let X be the number of heads in the 400 tossings.
- (a) Use Chebyshev's inequality to bound the probability that X is between 100 and 140.
- (b) Use Gaussian approximation to compute the probability that X is between 100 and 140.