

Prof. A. Suciu

Name: _____

MTH 1187–Probability

EXAM 2

Winter 2001

Instructions: This is an open-book, open-notes exam. There are 6 problems, worth 40 points in all. Put your name in the blanks above. **Show your work!** if there is not enough room, use the back page. Give all numerical answers as **fractions**, or as decimals, correct to at least **3 significant digits**. Remember: Probabilities are **never** greater than 1!

(1) [7 points] The student population at a large university is 55% male and 45% female. It is found that 15% of the men are left-handed, 90% of the women are right-handed, and nobody is ambidextrous.

(a) What proportion of all students is left-handed?

(b) If a student is right-handed, what is the probability that the student is female?

(2) [5 points] In this Probability class, a lottery game is set up. There are 35 students playing (independently), each one having a $1/50$ chance of winning. What is the chance that you win **and** nobody else wins?

- (3) [7 points] The lifetime of a certain type of lightbulb is a random variable X with probability density function and cumulative density function (measured in months of use) given by:

$$f_X(t) = 2te^{-t^2}, \quad F_X(t) = 1 - e^{-t^2}, \quad \text{for } t \geq 0.$$

- (a) What is the probability that a lightbulb lasts at least one month?
- (b) What is the probability that a lightbulb that has lasted half a month will expire by the age of two months?

- (4) [7 points] A **triple** in poker is a hand with pattern AAABC, where A, B, and C are from distinct kinds. The probability of being dealt a triple in a hand of poker is

$$\frac{\binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{1}^2}{\binom{52}{5}} = \frac{54,912}{2,598,960} = \frac{88}{4,165} = 0.0211285.$$

In a game of poker, you are dealt 100 hands.

- (a) Find the probability that you will be dealt **exactly** 3 triples.
- (b) Find the probability that you will be dealt 2 triples **or more**.

- (5) [7 points] Heights of children in a certain age group average 48.5 inches, with a standard deviation of 3 inches. Assume that the heights are normally distributed.
- (a) What proportion of children are between 45 and 50 inches tall?

(b) What is the 90th percentile of the children's heights?

- (6) [7 points] A professor teaches a Probability each Winter. For tests, he uses exams that he knows from past experience produce a normal grade distribution with mean 72 and standard deviation 8. His philosophy of grading is to impose standards that will yield, in the long run, 14% A's, 26% B's, 30% C's, 20% D's, and 10% F's.
- (a) What percentage of students score below 70?

- (b) Where should the cutoff be between the B's and the C's? (If you finish early, and still have energy, compute all the cutoff points between the grades.)