Instructor: Prof. A. Suciu

MTH 1124 **Calculus 2** **Winter 2001**

SOLUTIONS to QUIZ 7

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(1) [6 points] Consider de region in the plane, R, bounded by the x-axis, the y-axis, the graph of $y = e^x$, and the line $x = 1$. For each of the following, write a definite integral, or an expression involving a definite integral, which would yield the desired quantity. **DO NOT EVALUATE THESE INTEGRALS**.

(a) The volume of the solid obtained by revolving R around the x-axis.

$$\text{Volume} = \int_0^1 \pi\, e^{2x}\, dx$$

(b) The volume of the solid obtained by revolving R around the y-axis.

$$\text{Volume} = \pi \cdot e - \int_1^e \pi\, (\ln y)^2\, dy$$

(2) [4 points] Consider the curve $y = \sqrt{x^5}$. Write a definite integral that gives the arc length of the curve between $x = 0$ and $x = 2$. **DO NOT EVALUATE THIS INTEGRAL**.

$$\text{Length} = \int_0^2 \sqrt{1 + \left(\tfrac{5}{2}\, x^{3/2}\right)^2}\, dx$$

(3) [5 points] A rod of length 2 meters and density $\delta(x) = x$ kg/m is placed on the x-axis, with ends at $x = 0$ and $x = 2$. Find the coordinate of the center of mass of the rod.

$$\overline{x} = \frac{\int_0^2 x \cdot x dx}{\int_0^2 x dx} = \frac{\frac{x^3}{3}\Big|_0^2}{\frac{x^2}{2}\Big|_0^2} = \frac{4}{3}\ \text{m}$$

(4) [5 points] A square plate with side-length 2 is placed with its center at the origin. The density is given by $\delta(y) = y + 3$ kg/m^2, where y is the distance from the x-axis. Find the total mass of the plate.

$$\text{Mass} = \int_{-1}^1 2 \cdot (y + 3)\, dy = (y^2 + 6y)\Big|_{-1}^1 = 12\ \text{kg}$$