Prof. A. Suciu

MTH 1108 **Fundamentals of Integral Calculus** **Spring 1999**

QUIZ 4

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1. The height of a wave in a fish tank at a certain instant is given by $h(x) = \dfrac{1}{\sqrt{1-x^2}}$ meters, where x is the distance in meters from the left side of the tank. If the tank is half a meter long, find the average height of the wave at this instant. **(7)**

2. The region bounded by the curves $y = e^x + 1$, $x = 0$, and $x = 1$ is revolved around the x-axis. Find the volume of the resulting solid of revolution. **(7)**

3. A wave alongside a seawall has equation

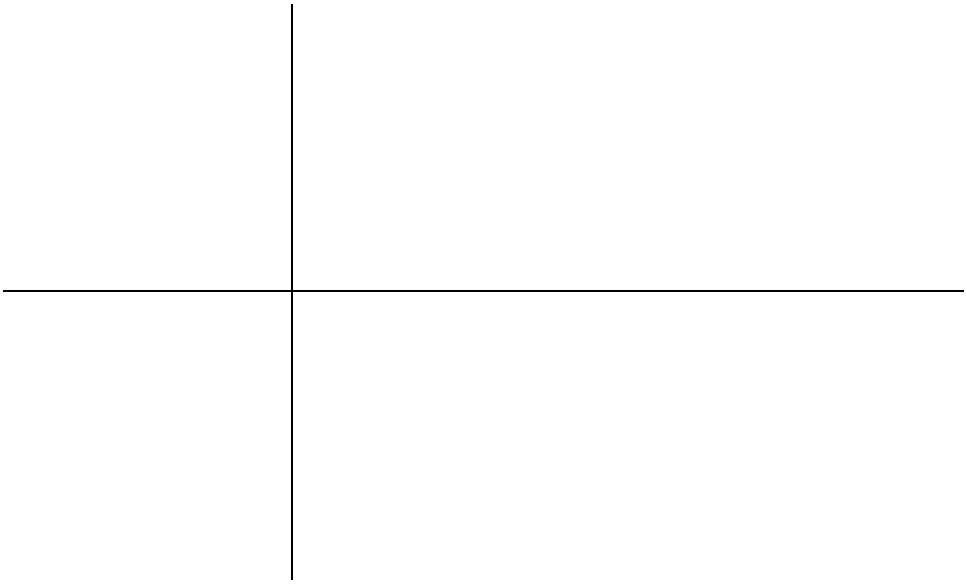
$$y(x) = -1 + 2\sin\left(\frac{\pi x}{4} - \frac{3\pi}{4}\right)$$

where x and y are in feet. **(16)**

(a) Find:

- The period (wavelength)
- The frequency
- The amplitude
- The vertical shift
- The horizontal shift

(b) Draw the graph of the wave. Show the period and amplitude of the function, and clearly indicate the scale on your graph.



(c) Find the slope of the wave at $x = 4$ feet.

Table of Derivatives

$$(fg)' = f'g + fg' \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \qquad f(g(x))' = f'(g(x)) \cdot g'(x)$$

$$(x^n)' = nx^{n-1} \qquad (e^x)' = e^x \qquad (\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x \qquad (\cos x)' = -\sin x \qquad (\tan x)' = \sec^2 x$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \qquad (\arctan x)' = \frac{1}{x^2+1}$$

Table of Antiderivatives

$$\int a\,dx = ax + C \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int e^{ax}\,dx = \frac{1}{a}\,e^{ax} + C \qquad (a \neq 0)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\,\cos(ax) + C \qquad \int \cos(ax)\,dx = \frac{1}{a}\,\sin(ax) + C \qquad (a \neq 0)$$

Properties of Integrals

$$\int (af(x) + bg(x))\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C, \qquad \text{where } F' = f$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^b f(t)dt = F(b) - F(a), \qquad \text{where } F' = f$$

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

Areas, Volumes, and Averages

Area between $y = f(x)$, $x = a$, and $x = b$: $\mathrm{A} = \displaystyle\int_a^b f(x)\,dx$.

Solid of revolution about x-axis, from a to b: $\mathrm{V} = \pi\displaystyle\int_a^b R^2\,dx$ or $\mathrm{V} = \pi\displaystyle\int_a^b (R^2 - r^2)\,dx$

Average value of $y = f(x)$, from a to b: $h = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$.