Prof. A. Suciu

MTH 1108 — **Fundamentals of Integral Calculus** — **Spring 1999**

MAKE-UP QUIZ 4

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1. Find the average value of the function $f(x) = \dfrac{x^2}{\sqrt{x^3+5}}$, for x between $x = 1$ and $x = 3$. **(7)**

2. The region bounded by the curves $y = \sqrt{x} + 1$, $x = 0$, and $x = 4$ is revolved around the x-axis. Find the volume of the resulting solid of revolution. **(7)**

3. A wave alongside a seawall has equation

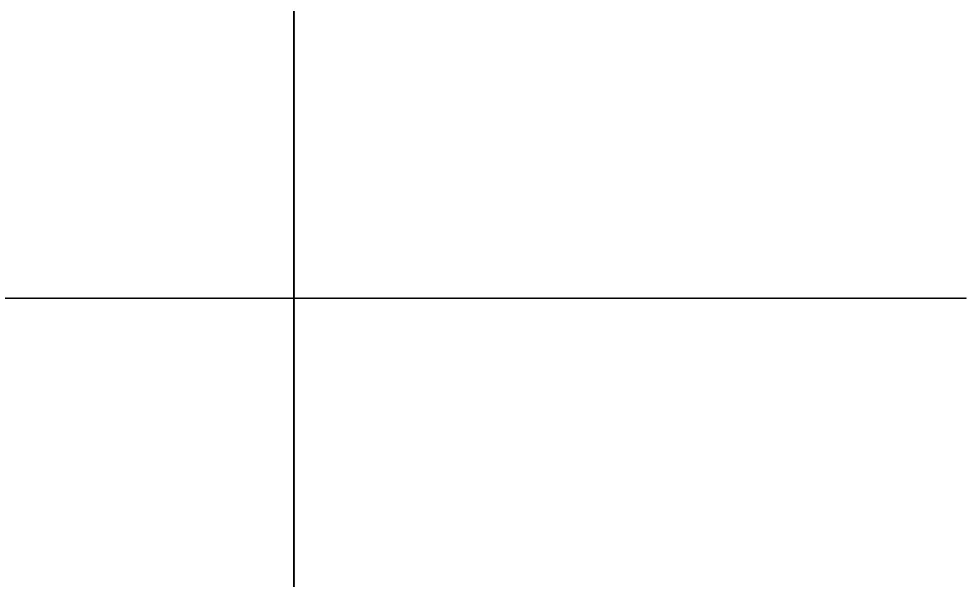
$$y(x) = \frac{1}{2} + \frac{3}{2}\sin\left(\frac{x}{4} + \frac{1}{8}\right)$$

where x and y are in feet. **(16)**

(a) Find:

- The period (wavelength)
- The frequency
- The amplitude
- The vertical shift
- The horizontal shift

(b) Draw the graph of the wave. Show the period and amplitude of the function, and clearly indicate the scale on your graph.



(c) Find the highest and the lowest points of the wave.

(d) Find the slope of the wave at $x = 8$ feet.

Table of Derivatives

$$(fg)' = f'g + fg' \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \qquad f(g(x))' = f'(g(x)) \cdot g'(x)$$

$$(x^n)' = nx^{n-1} \qquad (e^x)' = e^x \qquad (\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x \qquad (\cos x)' = -\sin x \qquad (\tan x)' = \sec^2 x$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \qquad (\arctan x)' = \frac{1}{x^2+1}$$

Table of Antiderivatives

$$\int a\,dx = ax + C \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int e^{ax}\,dx = \frac{1}{a}\,e^{ax} + C \quad (a \neq 0)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\,\cos(ax) + C \qquad \int \cos(ax)\,dx = \frac{1}{a}\,\sin(ax) + C \quad (a \neq 0)$$

Properties of Integrals

$$\int (af(x) + bg(x))\;dx = a\int f(x)\,dx + b\int g(x)\,dx$$

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C, \qquad \text{where } F' = f$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^b f(t)dt = F(b) - F(a), \qquad \text{where } F' = f$$

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

Areas, Volumes, and Averages

Area between $y = f(x)$, $x = a$, and $x = b$: $\mathrm{A} = \int_a^b f(x)\,dx$.

Solid of revolution about x-axis, from a to b: $\mathrm{V} = \pi\int_a^b R^2\,dx$ or $\mathrm{V} = \pi\int_a^b (R^2 - r^2)\,dx$

Average value of $y = f(x)$, from a to b: $h = \frac{1}{b-a}\int_a^b f(x)\,dx$.