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MTH 1108 **Fundamentals of Integral Calculus** **Spring 1999**

ANSWERS TO QUIZ 5

1. Solve the differential equation $yy' = e^{8x}$ by separating the variables. Then determine the solution $y = y(x)$ for which $y(0) = 3$.

$$y\frac{dy}{dx} = e^{8x} \implies \int y\,dy = \int e^{8x}\,dx + C \implies \frac{y^2}{2} = \frac{e^{8x}}{8} + C \implies y = \pm\frac{\sqrt{e^{8x}+K}}{2}.$$

$$y(0) = 3 \implies 3 = \pm\frac{\sqrt{1+K}}{2} \implies \text{sign is } + \text{ and } 1 + K = (2\cdot 3)^2 \implies K = 35.$$

Thus, the solution is $y(x) = \dfrac{\sqrt{e^{8x}+35}}{2}$.

2. Find all the values of k for which the function $y(x) = e^{kx}$ is a solution to the differential equation $y'' - 5y' + 6y = 0$.

$$y = e^{kx}, \qquad y' = ke^{kx}, \qquad y'' = k^2e^{kx}.$$

$$y'' - 5y' + 6y = k^2e^{kx} - 5ke^{kx} + 6e^{kx} = e^{kx}(k^2 - 5k + 6) = e^{kx}(k-2)(k-3)$$

$$y'' - 5y' + 6y = 0 \implies (k-2)(k-3) = 0 \implies k = 2 \text{ or } k = 3.$$

3. A glass of lemonade at 35°F is taken out of a refrigerator and brought into a room that has constant temperature 70°F. After 2 minutes, the temperature of the lemonade rises to 45°F. Suppose Newton's law of cooling applies.

(a) What differential equation describes the rate of warming of the lemonade?

Let $y = y(t)$ be the temperature of the lemonade at time t minutes after is was brought into the room. According to Newton's law of cooling, the rate of change of y is given by

$$y' = k(70 - y)$$

where k is a constant to be determined (see below).

(b) Find the temperature $y(t)$ of the lemonade at time t minutes after is was brought into the room.

$$y(t) = 70 + Ce^{-kt}$$

$$35 = y(0) = 70 + Ce^0 \implies C = -35$$

$$45 = y(2) = 70 - 35e^{-2k} \implies -25 = -35e^{-2k} \implies k = -\frac{\ln\left(\frac{5}{7}\right)}{2} \doteq 0.168236$$

Thus, $y(t) = 70 - 35e^{-0.168236\,t}$.

(c) What is the temperature of the lemonade, 5 minutes after is was brought into the room?

$$y(5) = 70 - 35e^{-0.168236\cdot 5} \doteq 54.908 \text{ (degrees Fahrenheit)}$$

(d) What is the rate of warming of the lemonade, 5 minutes after is was brought into the room?

$$y'(5) = k(70 - y(5)) \doteq 0.168236(70 - 54.908) \doteq 2.539 \text{ (degrees Fahrenheit per minute)}$$