**Instructor: Prof. A. Suciu** **Name:** ______________________

**MTH 1101** **Applications of Algebra** **Fall 2002**

# MIDTERM EXAM

**Instructions**: Put your name in the blanks above. Put your final answers to each question in the designated spaces on these pages. Show your work—if there is not enough room, use another sheet.

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**(1)** **SET UP** a linear program to solve the following problem. Be sure to identify the variables, **ALL** the constraints, and the objective function. **DO NOT SOLVE**. **[16 points]**

Joe's Scary Costume Shop makes two different types of Halloween masks: the scream mask and the evil clown mask. Each scream mask requires 9 ounces of rubber, 5 ounces of dye, and sells for \$13.50. Each evil clown mask requires 12 ounces of rubber, 4 ounces of dye, and sells for \$15.75. On the Wednesday before Halloween, Joe's Scary Costume Shop would like to make at least 80 masks. However, they would like to make no more than 50 scream masks, due to limited demand for that type of mask. Furthermore, they would like to make at least as many evil clown masks as scream masks. The Costume Shop has 600 ounces of rubber and 160 ounces of dye available. How many of each type of mask should they make in order to maximize sales?

**(2)** Use the addition method to find the point of intersection of the lines **[9 points]**

$$3x + 2y = 5$$
$$5x + 6y = -3$$

**DO NOT express your answer in digital form. DO NOT graph the lines.**

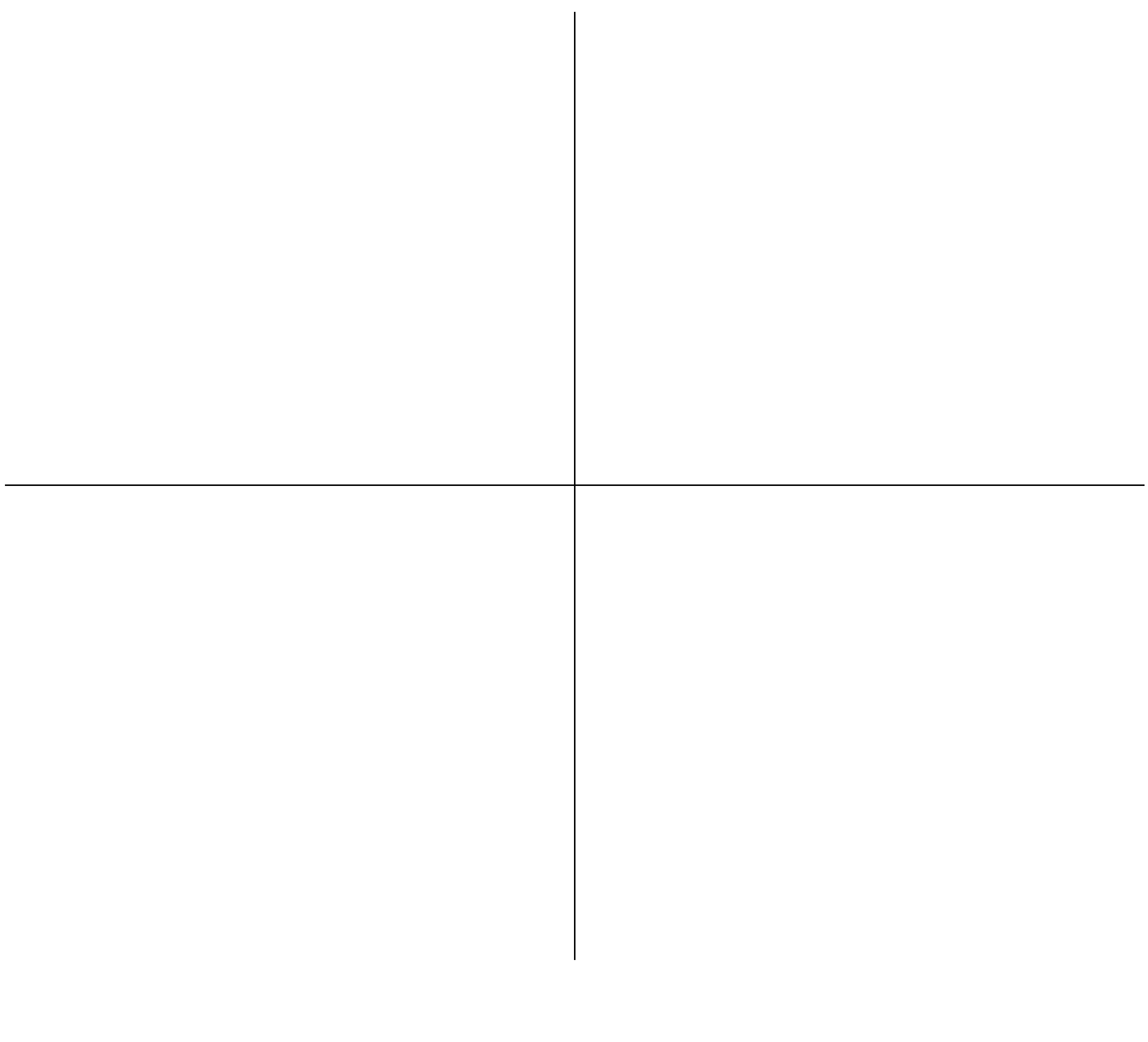
**(3)** Given the system of equations $\begin{cases} 7x - 4y = 6 \\ 8x - 5y = 7 \end{cases}$

(a) Express the system in matrix form. **[3 points]**

(b) Solve the system by using the inverse of the coefficient matrix. **[9 points]**

**(4)** Solve the following system of linear inequalities AND shade the region. **[12 points]**

$$x \geq 0, \qquad y < 3, \qquad 2x + y \geq 0$$



**(5)** Find $5A - 4B$, where $A = \begin{bmatrix} 2 & -1 & 3 & 5 \\ 1 & -4 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 & 2 & 0 \\ 3 & 2 & -5 & 1 \end{bmatrix}$. **[8 points]**

**(6)** Find the product $\begin{bmatrix} 4 & 7 \\ -6 & 1 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} -8 & 0 & 6 \\ 4 & 7 & -3 \end{bmatrix}$

**Show the work, not just the calculator result.** [**12 points**]

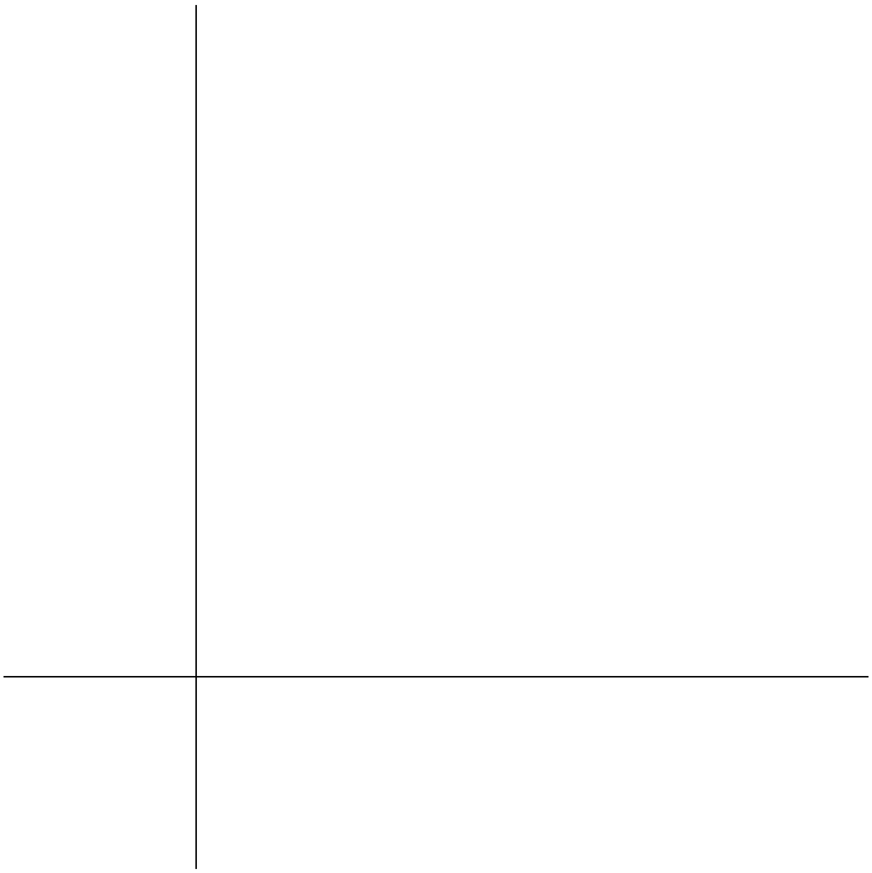
**(7) SOLVE** the following linear programming problem:

Maximize and minimize $F = 2x + 3y$

subject to the constraints: $x \geq 2, \quad x \leq 8, \quad y \geq 1, \quad y \leq 4, \quad x + 3y \geq 9$

(a) Shade the feasible region, and determine the coordinates of all the corner points.

**[12 points]**



(b) Determine the solution to the problem. **[7 points]**

**Answer:**

- The maximum value of $F$ is __________, and it occurs at the point (_____ , _____).
- The minimum value of $F$ is __________, and it occurs at the point (_____ , _____).

**(8)** The message $-14,\ -26,\ 23,\ 38,\ -10,\ -17,\ 48,\ 78$
was encoded using the matrix

$$M = \begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}$$

and the coding scheme

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ | $I$ | $J$ | $K$ | $L$ | $M$ | $N$ | $O$ | $P$ |
| 1 | −1 | 2 | −2 | 3 | −3 | 4 | −4 | 5 | −5 | 6 | −6 | 7 | −7 | 8 | −8 |
| $Q$ | $R$ | $S$ | $T$ | $U$ | $V$ | $W$ | $X$ | $Y$ | $Z$ | blank | ‘ | , | . | ! | ? |
| 9 | −9 | 10 | −10 | 11 | −11 | 12 | −12 | 13 | −13 | 14 | −14 | 15 | −15 | 16 | −16 |

(a) What matrix is needed for decoding the message? **[4 points]**

(b) What is the message? **[8 points]**

**The message is:** ______________________________