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MTH U576 **Rings and Fields** Spring 2007

QUIZ 6

(1) (6 points) Let V be a vector space over a field F. Let $v_1, \ldots, v_n$ and w be elements of V. For each of the following questions, answer **Yes** if the conclusion always holds, **No** if the conclusion never holds, and **Maybe** if the conclusion sometimes holds, and sometimes not.

(a) Suppose $\{v_1, \ldots, v_n\}$ spans V. Does $\{v_1, \ldots, v_{n-1}\}$ span V?

(b) Suppose $\{v_1, \ldots, v_n\}$ is linearly independent. Is $\{v_1, \ldots, v_{n-1}\}$ linearly independent?

(c) Suppose $\{v_1, \ldots, v_n\}$ is a basis for V. Is $\{v_1, \ldots, v_{n-1}\}$ a basis for V?

(d) Suppose $\{v_1, \ldots, v_n\}$ spans V. Does $\{v_1, \ldots, v_n, w\}$ span V?

(e) Suppose $\{v_1, \ldots, v_n\}$ is linearly independent. Is $\{v_1, \ldots, v_n, w\}$ linearly independent?

(f) Suppose $\{v_1, \ldots, v_n\}$ is a basis for V. Is $\{v_1, \ldots, v_n, w\}$ a basis for V?

(2) (4 points) Consider the field $\mathbb{R}$, viewed as a vector space over $\mathbb{Q}$.

(a) Is the subset $\{1, \sqrt{3}\}$ linearly independent (over $\mathbb{Q}$)?

(b) Is $\sqrt{2}$ a linear combination of 1 and 3 (over $\mathbb{Q}$)?

(c) Does the subset $\{1, \sqrt{3}\}$ span $\mathbb{R}$?

(3) (6 points) Let $F \subset K$ be an extension of fields, and let $u \in K$.

(a) Suppose u is algebraic over F. Show that $u + 1$ is algebraic over F.

(b) Suppose u is transcendental over F. Show that $u + 1$ is transcendental over F.

(c) Show that $F(u) = F(u + 1)$.

(4) (4 points) Find the minimal polynomial of $\sqrt{1 + \sqrt{3}}$ over $\mathbb{Q}$.