Instructor: Prof. A. Suciu Name: _____

MTH U576 **Rings and Fields** Spring 2007

QUIZ 5

(1) (3 points) How many congruence classes modulo $x^2 + x + 2$ are there in $\mathbb{Z}_3[x]$? List them all.

(2) (7 points) Consider the congruence-class ring $S = \mathbb{Z}_2[x]/(x^2 + x)$.

(a) Write out the addition and multiplication tables for S.

(b) What are the units (if any) in S?

(c) What are the zero-divisors (if any) in S?

(3) (5 points) Which of the following congruence-class rings is a field? Explain.
(a) $\mathbb{Q}[x]/(x^3 - 2x^2 + 2x - 2)$

(b) $\mathbb{Q}[x]/(x^3 - 2x^2 + x - 2)$

(4) (5 points) If $p(x)$ is an irreducible quadratic polynomial in $F[x]$, show that $F[x]/(p(x))$ contains all the roots of $p(x)$.