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MTH U576 **Rings and Fields** Spring 2007

QUIZ 4

(1) (6 points) Consider the polynomial

$$f(x) = 4x^4 + 8x^3 - 7x^2 - 11x + 6$$

(a) What are the rational roots of f allowed by the Rational Root Test? [List **all** the possibilities.]

(b) Use the above information to factor f as a product of irreducible polynomials.

(2) (4 points) Use Eisenstein's Criterion to show that $\sqrt{10}$ is irrational.

[Indicate which polynomial and which prime is used, and how the Criterion applies.]

(3) (5 points) Use Eisenstein's Criterion to show that the following polynomial is irreducible in $\mathbb{Q}[x]$.

[Indicate which prime is used, and how the Criterion applies. You will need to perform a preliminary change of variable, of the form $x \mapsto x + c$, for some suitable constant c.]

$$f(x) = x^4 - x^3 + x^2 - x + 1$$

(4) (5 points) Show that the following polynomial $f(x)$ is irreducible in $\mathbb{Q}[x]$, by finding a prime p so that $f(x)$ is irreducible in $\mathbb{Z}_p[x]$.

$$f(x) = x^4 + 4x^3 + 8x^2 + 3x + 5$$