Prof. Alexandru Suciu

MTH U565 **TOPOLOGY** **Spring 2008**

HOMEWORK 7

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1. Let X be a topological space, and let $f\colon X \to S^n$ be a continuous map to the n-sphere ($n \geq 0$). Show that if f is not surjective, then f is homotopic to a constant map. [Hint: Use Proposition 6.5.]

2. Let $f\colon S^1 \to S^1$, $f(x, y) = (-x, -y)$. Show that f is homotopic to the identity map. What is $\deg(f)$?

3. Let $f\colon S^1 \to S^1$, $f(x, y) = (x, -y)$. What is $\deg(f)$?

4. Represent the circle S^1 as the set of complex numbers z of absolute value 1. Consider the maps $f\colon S^1 \to S^1$ and $g\colon S^1 \to S^1$ given by $f(z) = z^n$ and $g(z) = 1/z^n$. Compute $\deg(f)$ and $\deg(g)$.

5. Let A be a 3×3 matrix. Suppose all entries of A are real and non-negative, and that $\det(A) \neq 0$. Show that A has a positive real eigenvalue.