

HOMEWORK 6

Definition 1. Let X be a topological space, A a subspace, and $i: A \rightarrow X$ the inclusion map. A continuous map $r: X \rightarrow A$ is called a *retraction* if $r \circ i = \text{id}_A$; that is, $r(a) = a$, for every $a \in A$.

Definition 2. A subspace $A \subset X$ is called a *retract* of X if there is a retraction $r: X \rightarrow A$.

Definition 3. A subspace $A \subset X$ is called a *deformation retract* of X if there is a retraction $r: X \rightarrow A$ with the property that $i \circ r \simeq \text{id}_X$.

1. If A is a retract of a contractible space X , then A is a deformation retraction of X .
2. Show that a retract of a contractible space is contractible.
3. Find a retraction from the punctured plane $\mathbb{R}^2 \setminus \{0\}$ to the unit circle S^1 .
4. Show that the closed interval $[0, 1]$ is a deformation retract of the real line \mathbb{R} .
5. Prove that a discrete space consisting of m points is homotopy equivalent to a discrete space consisting of n points if, and only if, $m = n$.