Prof. Alexandru Suciu

MTH U565 **TOPOLOGY** **Spring 2008**

HOMEWORK 5

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1. Let $X = \{a, b, c\}$, with open sets $\emptyset$, $\{b\}$, $\{a, b\}$, $\{b, c\}$, $\{a, b, c\}$, and let $Y = \{0, 1\}$, with open sets $\emptyset$, $\{0\}$, $\{0, 1\}$. List all the open sets in the product topology for $X \times Y$. Is $X \times Y$ compact? connected? Hausdorff?

2. Let $f \colon X \to Y$ be a continuous function. Suppose X is path-connected. Show that $f(X)$ is also path-connected. As a corollary, show that path-connectedness is a topological invariant.

3. Let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on the set X. Suppose $\mathcal{T}' \supset \mathcal{T}$. What does connectedness of X under one of these topologies imply about connectedness under the other?

4. Let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on the set X. Suppose $\mathcal{T}' \supset \mathcal{T}$. What does compactness of X under one of these topologies imply about compactness under the other?

5. Let $A_1, \dots, A_n$ be compact subspaces of a space X. Show that

$$A = \bigcup_{i=1}^{n} A_i$$

is also compact.