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MTH U565 **TOPOLOGY** **Spring 2008**

MIDTERM EXAM

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1. Let $f\colon X \to Y$ be an open map. Let A be an open subset of X, and let $g\colon A \to f(A)$ be the map obtained by restricting f to A. Show that g is an open map.

2. Let $p\colon X \to Y$ be a continuous map. Suppose there is a continuous map $f\colon Y \to X$ such that $p \circ f$ equals the identity map of Y. Show that p is quotient map.

3. Let X and Y be path-connected spaces. Is the product space, $X \times Y$, path-connected? (If yes, give an argument why; if not, give a counterexample.)

4. Let X be a topological space. Suppose X contains an infinite, closed, discrete subspace. Show that X is *not* compact.

5. Let $X = [-1, 2]/(0, 1)$ be the quotient space of the closed interval $[-1, 2]$ by the open interval $(0, 1)$. Determine whether the space X is (a) compact; (b) connected; (c) Hausdorff.

6. Let $X = \{1, 2, 3, 4\}$, endowed with the topology

$$\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}.$$

Let $Y = \{a, b, c\}$, and $p\colon X \to Y$ the function sending $1 \mapsto a$, $2 \mapsto b$, $3 \mapsto b$, $4 \mapsto c$. Find the quotient topology on Y defined by the map p.

7. Let $X = [1, 3)$, $Y = [-1, 2]$, and $Z = (-2, 0]$. For each pair of these spaces, determine whether they are homeomorphic. (If they are, provide an explicit homeomorphism; if they are not, explain why.)