Prof. Alexandru Suciu

MTH U565 TOPOLOGY Spring 2008

# FINAL EXAM

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**1.** Let $X$ and $Y$ be topological spaces, and $p_1 \colon X \times Y \to X$ the first-coordinate projection map. Prove or disprove the following assertions.

(a) $p_1$ is a continuous map.

(b) $p_1$ is an open map.

(c) $p_1$ is a closed map.

**2.** Let $\mathbb{RP}^2$ be the real projective plane, defined as the quotient space of $\mathbb{R}^3 \setminus \{0\}$ by the equivalence relation $\vec{x} \sim \vec{y}$ if $\vec{x} = \lambda \vec{y}$, for some $\lambda \in \mathbb{R} \setminus \{0\}$. Show that:

(a) $\mathbb{RP}^2$ is homeomorphic to the quotient space of $S^2$, obtained by identifying antipodal points.

(b) $\mathbb{RP}^2$ is compact.

(c) $\mathbb{RP}^2$ is connected.

**3.** Consider the unit circle $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. Let $f \colon S^1 \to S^1$ be the map defined by $f(x, y) = (-x, y)$. What is the degree of $f$?

**4.** (a) For each integer $n$, find a *connected* finite simplicial complex with Euler characteristic equal to $n$.

(b) Which integers $n$ can occur as the Euler characteristic of a connected, 0-dimensional, finite simplicial complex?

(c) Which integers $n$ can occur as the Euler characteristic of a connected, 1-dimensional, finite simplicial complex?

**5.** (a) Show that every triangulation of the Klein bottle has at least 7 vertices.

(b) Find a triangulation of the Klein bottle with exactly 8 vertices.

(c) Compute the Euler characteristic of the Klein bottle, using this triangulation.

**6.** Let $K$ be the simplicial complex on vertex set $\{1, 2, 3, 4, 5, 6\}$, with maximal simplices $\{123, 124, 134, 234, 25, 26, 36, 56\}$.

(a) Draw a picture of the geometric realization of $K$.

(b) Write down the simplicial chain complex $C_*(K) = (C_n(K), \partial_n)_{n \geq 0}$, with $\mathbb{Z}_2$ coefficients.

(c) Compute the homology groups of $K$ (with $\mathbb{Z}_2$ coefficients).

(d) What is the Euler characteristic of $K$?