Prof. Alexandru Suciu

MTH U565 **TOPOLOGY** **Spring 2006**

MIDTERM EXAM

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1. Let $f\colon X \to Y$ be a continuous surjection, and suppose f is a closed map (i.e., it takes closed sets in X to closed sets in Y). Let $g\colon Y \to Z$ be a function so that $g \circ f\colon X \to Z$ is continuous. Show that g is continuous.

2. Let X be a space. Show that X is Hausdorff if, and only if, the diagonal $\Delta := \{(x,x) \mid x \in X\}$ is a closed subspace of $X \times X$.

3. Let $X = [0,1]/(\frac{1}{4},\frac{3}{4})$ be the quotient space of the unit interval, where the open interval $(\frac{1}{4},\frac{3}{4})$ is identified to a single point. Show that X is not a Hausdorff space.

4. Let X be a Hausdorff space. Suppose A is a compact subspace, and $x \in X \setminus A$. Show that there exist disjoint open sets U and V containing A and x, respectively.

5. Let $p\colon X \to Y$ be a quotient map. Suppose Y is connected, and, for each $y \in Y$, the subspace $p^{-1}(\{y\})$ is connected. Show that X is connected.

6. Let X be a discrete topological space, and let $\sim$ be an equivalence relation on X. Prove that $X/{\sim}$, endowed with the quotient topology, is also a discrete space.