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MTH U565 TOPOLOGY Spring 2006

FINAL EXAM

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1. (a) Suppose Y is a closed subspace of a topological space X, and A is a closed subspace of Y. Show that A is a closed subspace of X.

(b) Suppose A is a closed subspace of X, and B is a closed subspace of Y. Show that $A \times B$ is a closed subspace of $X \times Y$.

2. Let Y be a compact space, and X an arbitrary space. Show that the first-coordinate projection map, $p_1 \colon X \times Y \to X$, is a closed map.

3. Let X be a topological space, and $f \colon X \to S^n$ a continuous map. Show that, if f is not surjective, then f is homotopic to a constant map.

4. (a) Show that $\mathbb{R}^2 \setminus \{n \text{ points}\}$ is homotopy equivalent to a bouquet of circles. How many circles are there in this bouquet?

(b) Show that $S^2 \setminus \{n \text{ points}\}$ is homotopy equivalent to a bouquet of circles. How many circles are there in this bouquet?

5. Let K and L be two (finite) simplicial complexes. Let A be a sub-simplicial complex of both K and L, and let $K \cup_A L$ be the simplicial complex obtained by gluing K and L along A. Prove that:

$$\chi(K \cup_A L) = \chi(K) + \chi(L) - \chi(A).$$

6. Let K be the simplicial complex consisting of a the boundary of a tetrahedron, with a (hollow) triangle attached to a vertex. In other words, K is the simplicial complex on vertex set $\{1, 2, 3, 4, 5, 6\}$, with maximal simplices $123, 124, 134, 234, 45, 46, 56$.

(a) Write down the simplicial chain complex $C_*(K) = (C_n(K), \partial_n)_{n \geq 0}$, with $\mathbb{Z}_2$ coefficients.

(b) Compute the homology groups of K (with $\mathbb{Z}_2$ coefficients).