

1. 9 points(a) Find the least squares solution  $\vec{x}^*$  of the inconsistent system  $A\vec{x} = \vec{b}$ , where

$$A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

(b) Use your answer to part (a) to find the projection of  $\vec{b}$  onto  $\text{im } A$ .(c) Determine the error  $\|\vec{b} - A\vec{x}^*\|$ .

2. 6 points Let  $A, B, C$  be three square matrices, with  $\det A = -1$ ,  $\det B = 0$ ,  $\det C = 3$ . In the following, circle the correct answer.

- |                      |             |           |                              |
|----------------------|-------------|-----------|------------------------------|
| • Is $A$ invertible? | Yes, always | No, never | Sometimes yes, sometimes not |
| • Is $B$ invertible? | Yes, always | No, never | Sometimes yes, sometimes not |
| • Is $C$ invertible? | Yes, always | No, never | Sometimes yes, sometimes not |
| • Is $A$ orthogonal? | Yes, always | No, never | Sometimes yes, sometimes not |
| • Is $B$ orthogonal? | Yes, always | No, never | Sometimes yes, sometimes not |
| • Is $C$ orthogonal? | Yes, always | No, never | Sometimes yes, sometimes not |

3. 4 points Let  $A$  and  $B$  be two  $4 \times 4$  matrices, with  $\det A = -2$  and  $\det B = 5$ . Compute:

(a)  $\det(3B) =$

(b)  $\det(A^5) =$

(c)  $\det((B^T)^{-1}) =$

(d)  $\det(A \cdot B^2 \cdot A^{-1} \cdot B^T) =$

4. 6 points Consider a  $3 \times 3$  matrix  $A$  with rows  $\vec{v}_1, \vec{v}_2, \vec{v}_3$ . Suppose  $\det(A) = 7$ . Compute:

(a)  $\det \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 - 3\vec{v}_1 \\ \vec{v}_3 + \vec{v}_1 \end{bmatrix}$

(b)  $\det \begin{bmatrix} \vec{v}_3 \\ \vec{v}_2 \\ \vec{v}_1 \end{bmatrix}$

(c)  $\det \begin{bmatrix} 3\vec{v}_1 \\ \vec{v}_3 + \vec{v}_1 \\ \vec{v}_2 \end{bmatrix}$