Prof. Alexandru Suciu

MTH U371 **LINEAR ALGEBRA** **Spring 2006**

QUIZ 5

1. $\boxed{\text{8 points}}$ Apply the Gram-Schmidt process to the vectors $\vec{v}_1 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, and write the result in the form $A = Q \cdot R$.

2. $\boxed{\text{6 points}}$ Consider the vectors $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 5 \\ 5 \end{bmatrix}$.

(a) Find the matrix of the orthogonal projection onto the line L in $\mathbb{R}^3$ spanned by $\vec{v}$.

(b) Find the projection of $\vec{w}$ onto the line L.

3. $\boxed{\text{5 points}}$ Is there an orthogonal transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that $T\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}-3\\1\end{bmatrix}$ and $T\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}-2\\-1\end{bmatrix}$? Justify your answer, one way or the other.

4. $\boxed{\text{6 points}}$ Let A be an arbitrary $n \times n$ matrix. and let Q be an orthogonal $n \times n$ matrix. For each of the following questions, answer: "Yes, always," or "Sometimes yes, sometimes not," or "No, never." Justify your answer, as much as possible.

(a) The matrix $AA^\top$ is symmetric.

(b) The matrix $AA^\top$ is invertible.

(c) The matrix $AA^\top$ is orthogonal.

(d) The matrix $Q^\top$ is symmetric.

(e) The matrix $Q^\top$ is invertible.

(f) The matrix $Q^\top$ is orthogonal.