

1. 3 points Sketch the image of the unit square under the linear transformation

$$T(\vec{x}) = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \vec{x}$$

2. 8 points Find the matrices of the following linear transformations:

(a) $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, a clockwise rotation of 60° , followed by a dilation by a factor of 2.

(b) $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, the reflection in the line $y = -x$.

(c) $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, the reflection in the y - z -plane, followed by a scaling by a factor of $1/2$.

(d) $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, the projection onto the x - z -plane.

3. 8 points Let $A = \begin{bmatrix} 0 & 0 & 1 & 2 & 3 \\ 1 & -1 & 0 & 4 & 6 \\ 2 & -2 & 1 & 10 & 15 \end{bmatrix}$.

- (a) Find the row-reduced echelon form of A .
- (b) Find vectors that span the image of A . Give as few vectors as possible.
- (c) Find vectors that span the kernel of A .

4. 6 points Let V be the subspace of \mathbb{R}^3 defined by the equation $3x_1 + 5x_2 - x_3 = 0$.

- (a) Express V as the kernel of a matrix A .

- (b) Express V as the image of a matrix B .