MTH U371

1. 2 points Find the matrix A of the linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ given by

 $y_1 = -x_1 + 3x_2 + 2x_3$ $y_2 = -5x_2 + 2x_3 - x_4$ $y_3 = 7x_1 + x_2 - 4x_3 + x_4$

2. 4 points Consider the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$, where

$$T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}, \qquad T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}, \qquad T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}2\\-5\end{bmatrix}.$$

(a) Find the matrix A of T.

(b) Compute
$$T \begin{bmatrix} 4\\ -3\\ 7 \end{bmatrix} =$$

3. 4 points Let

$$A = \begin{bmatrix} -1 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix}.$$

Decide whether the following products are defined or not. If they are, compute them: $A \cdot B, \quad B \cdot A, \quad A \cdot C, \quad C \cdot A, \quad B \cdot C, \quad C \cdot B.$ 4. 5 points Use Gaussian elimination to find the inverse of following matrix. Indicate for each step which row operation you use.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

5. 5 points For which choices of the constant k is the following matrix invertible?

	[1	1	1
A =	k	0	-2
	1	k	3

6. 5 points Find the matrix A of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\begin{bmatrix} 6\\5 \end{bmatrix} = \begin{bmatrix} 7\\-6 \end{bmatrix}, \quad T\begin{bmatrix} 4\\3 \end{bmatrix} = \begin{bmatrix} -2\\1 \end{bmatrix}.$$