1. 2 points Find the matrix $A$ of the linear transformation $T: \mathbb{R}^{4} \rightarrow \mathbb{R}^{3}$ given by

$$
y_{1}=-x_{1}+3 x_{2}+2 x_{3}
$$

$$
y_{2}=-5 x_{2}+2 x_{3}-x_{4}
$$

$$
y_{3}=7 x_{1}+x_{2}-4 x_{3}+x_{4}
$$

2. 4 points Consider the linear transformation $T: \mathbb{R}^{3} \rightarrow \mathbb{R}^{2}$, where

$$
T\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right]=\left[\begin{array}{l}
1 \\
2
\end{array}\right], \quad T\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right]=\left[\begin{array}{l}
3 \\
4
\end{array}\right], \quad T\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]=\left[\begin{array}{c}
2 \\
-5
\end{array}\right] .
$$

(a) Find the matrix $A$ of $T$.
(b) Compute $T\left[\begin{array}{c}4 \\ -3 \\ 7\end{array}\right]=$
3. 4 points Let

$$
A=\left[\begin{array}{cc}
-1 & 4
\end{array}\right], \quad B=\left[\begin{array}{cc}
5 & -1 \\
2 & 3
\end{array}\right], \quad C=\left[\begin{array}{lll}
2 & 0 & 1 \\
0 & 2 & 3
\end{array}\right]
$$

Decide whether the following products are defined or not. If they are, compute them:

$$
A \cdot B, \quad B \cdot A, \quad A \cdot C, \quad C \cdot A, \quad B \cdot C, \quad C \cdot B .
$$

4. 5 points Use Gaussian elimination to find the inverse of following matrix. Indicate for each step which row operation you use.

$$
A=\left[\begin{array}{lll}
1 & 2 & 0 \\
3 & 4 & 0 \\
1 & 1 & 1
\end{array}\right]
$$

5. 5 points For which choices of the constant $k$ is the following matrix invertible?

$$
A=\left[\begin{array}{ccc}
1 & 1 & 1 \\
k & 0 & -2 \\
1 & k & 3
\end{array}\right]
$$

6. 5 points Find the matrix $A$ of the linear transformation $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ with

$$
T\left[\begin{array}{l}
6 \\
5
\end{array}\right]=\left[\begin{array}{c}
7 \\
-6
\end{array}\right], \quad T\left[\begin{array}{l}
4 \\
3
\end{array}\right]=\left[\begin{array}{c}
-2 \\
1
\end{array}\right] .
$$

