QUIZ 1

1. 6 points Use Gauss-Jordan elimination to solve the following system of equations. **Indicate for each step which row operation you use**. Carry out the elimination **all the way** (to rref form) before solving.

$$
\begin{array}{rcrcrcr}
x & - & 2y & + & z & = & 0 \\
 & & 2y & - & 8z & = & 8 \\
-4x & + & 5y & + & 9z & = & -9
\end{array}
$$

2. 8 points Consider the following system of linear equations:

$$
\begin{array}{rcrcrcr}
x_1 - 2x_2 - x_3 + 3x_4 &=& 0 \\
-2x_1 + 4x_2 + 5x_3 - 5x_4 &=& 3 \\
3x_1 - 6x_2 - 6x_3 + 8x_4 &=& -3
\end{array}
$$

Identify which variables are leading and which are free. Write down the general solution of the system, in **vector form**.

3. $\boxed{\text{3 points}}$ The reduced row echelon forms of the augmented matrices of 3 systems are given below. In each case, indicate the rank of the matrix of coefficients (to the left of the dotted lines), and the number of solutions of the system (you need **not** write down the solutions.)

$$\left[\begin{array}{ccc:c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc:c} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

4. $\boxed{\text{8 points}}$ For a linear system $A\vec{x} = \vec{b}$, you are given either the rank of the coefficient matrix A, or the rank of the augmented matrix $[\,A : \vec{b}\,]$. In each case, indicate what are the possible values for the missing rank, and state whether the system could have no solution, one solution, or infinitely many solutions. (There may be more than one possibility for some systems.)

	Number of equations	Number of unknowns	Rank of A	Rank of $[A : \vec{b}]$	Number of solutions
a)	5	4	4		
b)	4	5		3	
c)	4	5	4		
d)	5	4		5	