Prof. A. Suciu · MTH U371–LINEAR ALGEBRA · Spring 2005

PRACTICE QUIZ 5

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1. Use Gauss-Jordan elimination to find the determinant of the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -2 \\ -1 & 2 & 1 & 6 \\ -2 & 6 & 10 & 33 \\ 2 & -2 & 5 & 10 \end{bmatrix}$.

2. Let A and B be two 5×5 matrices, with $\det A = 0$ and $\det B = -3$.

(a) Is A invertible? Why, or why not?

(b) Is A orthogonal? Why, or why not?

(c) Is B invertible? Why, or why not?

(d) Is B orthogonal? Why, or why not?

(e) Compute $\det(B \cdot A \cdot B)$.

(f) Compute $\det(B^{\top})^3$.

(g) Compute $\det(2B)$.

3. Find a 2×2 matrix A such that $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -5 \end{bmatrix}$ are eigenvectors of A, with eigenvalues -7 and 3, respectively.

4. A 4×4 matrix A has eigenvalues $\lambda_1 = -3$, $\lambda_2 = -2$, $\lambda_3 = 1$, $\lambda_4 = 4$.

(a) What is the characteristic polynomial of A?

(b) Compute $\operatorname{tr}(A)$ and $\det(A)$.

(c) What are the eigenvalues of A^2?

(d) Compute $\operatorname{tr}(A^2)$ and $\det(A^2)$.

(e) Compute $\det(A + 2I_4)$.

(f) Is A invertible? If yes, compute $\det(A^{-1})$. If not, explain why not.

(g) Is A diagonalizable? If yes, compute its diagonalization D. If not, explain why not.

5. Let $A = \begin{bmatrix} 4 & -7 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Find the eigenvalues of A.

(b) Find a basis for each eigenspace of A.

(c) Find a diagonal matrix D and an invertible matrix S such that $A = S \cdot D \cdot S^{-1}$.

6. A 2×2 matrix A has first row $\begin{bmatrix} -2 & 5 \end{bmatrix}$ and eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$.

(a) Find A.

(b) What are the eigenvalues of A^{-1}?

(c) Compute $\det(A^{-1} + I)$, where I is the identity 2×2 matrix. Explain your result.

7. A 4×4 matrix has eigenvalues $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3, \lambda_4 = 4$.

(a) Find the eigenvalues of A^2.

(b) Find the trace of A^2.

(c) Find the determinant of A^2.

8. Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) Find a basis for each eigenspace of A.

(d) Find a diagonal matrix D and an invertible matrix S such that $A = S \cdot D \cdot S^{-1}$.

9. Let A be a 3×3 matrix, with eigenvalues $\lambda_1 = -2$, $\lambda_2 = 0$, $\lambda_3 = 5$.

(a) Compute $\operatorname{tr}(A)$ and $\det(A)$.

(b) Is A invertible? Explain your answer.

(c) Is A diagonalizable? Explain your answer.

(d) Compute $\operatorname{tr}(A^3)$ and $\det(A^3)$.

10. Let A and B be two 3×3 matrices, with $\det A = -2$ and $\det B = 0$.

(a) Is A invertible? If yes, compute $\det(A^{-1})$. If not, say so.

(b) Is B invertible? If yes, compute $\det(B^{-1})$. If not, say so.

(c) Compute $\det(4A)$.

(d) Compute $\det(A^4)$.

11. Which of the following 2×2 matrices is similar to the matrix $D = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$?

$$A_1 = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 7 & 1 \\ 4 & 2 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 6 & -2 \\ 2 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} 2 & 1 \\ 0 & 5 \end{bmatrix}, \quad A_6 = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}.$$