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MTH U371 **LINEAR ALGEBRA** **Spring 2005**

QUIZ 4

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1. [8 points] Apply the Gram-Schmidt process to the vectors $\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$, and write the result in the form $A = Q \cdot R$.

2. 7 points

Consider the vectors $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 4 \\ 1 \\ 3 \\ 1 \end{bmatrix}$.

(a) Find the matrix of the orthogonal projection onto the line L in $\mathbb{R}^4$ spanned by $\vec{v}$.

(b) Find the projection of $\vec{w}$ onto the line L.

3. 6 points

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ be a 3×3 matrix.

(a) Is the matrix $B = A^\top A A^\top$ symmetric? Justify your answer.

(b) Is the matrix $B = 2A + 2A^\top$ symmetric? Justify your answer.

(c) Suppose A is orthogonal. What is A^{-1}?

4. 9 points

(a) Find the least squares solution $\vec{x}^*$ of the inconsistent system $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(b) Use your answer to part (a) to find the projection of $\vec{b}$ onto $\operatorname{im} A$.

(c) Determine the error $||\vec{b} - A\vec{x}^*||$.