MTH U345 **Ordinary Differential Equations** **Fall 2008**

Solutions to Quiz 5

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1. Find the Laplace transforms $F(s)$ of the following functions $f(t)$:

(a) $f(t) = \begin{cases} 0, & t < 3 \\ t^2 - 6t + 1, & t \geq 3 \end{cases}$

$$f(t) = u_3(t)((t-3)^2 - 8) \quad \longrightarrow \quad F(s) = e^{-3s}\left(\frac{2}{s^3} - \frac{8}{s}\right)$$

(b) $f(t) = e^{-4t}\delta_3(t) - e^{2t-2}u_1(t)$

$$F(s) = e^{-3(s+4)} - e^{-s}\frac{1}{s-2}$$

2. Find the inverse Laplace transform $f(t)$ of the following functions $F(s)$:

(a) $F(s) = \dfrac{4s-1}{s^2 - 4s + 13}$

$$F(s) = 4\frac{s-2}{(s-2)^2 + 3^2} + \frac{7}{3}\frac{3}{(s-2)^2 + 3^2} \quad \longrightarrow \quad f(t) = e^{2t}\left(4\cos(3t) + \frac{7}{3}\sin(3t)\right)$$

(b) $F(s) = \dfrac{4e^{-s}}{s^2 + 6s + 5}$

$$F(s) = e^{-s}\left(\frac{1}{s+1} - \frac{1}{s+5}\right) \quad \longrightarrow \quad f(t) = u_1(t)\left(e^{-(t-1)} - e^{-5(t-1)}\right)$$

3. Consider the initial value problem

$$y'' - 3y' + 2y = 1 + \sin(5t), \quad y(0) = -4, \quad y'(0) = 6$$

Determine the Laplace transform $Y(s)$ of the solution $y(t)$.

$$Y(s) = \frac{\frac{1}{s} + \frac{5}{s^2+25} - 4s + 18}{s^2 - 3s + 2} = \frac{-4s^4 + 18s^3 - 99s^2 + 455s + 25}{s(s-1)(s-2)(s^2+25)}$$

4. Solve the IVP: $\quad y'' = u_3(t), \quad y(0) = 0, \quad y'(0) = 0.$

$$Y(s) = \mathcal{L}[y(t)] \quad \longrightarrow \quad Y(s) = \frac{e^{-3s}}{s^3} \quad \longrightarrow \quad y(t) = \frac{1}{2}u_3(t)(t-3)^2$$