Name: ____________________

MTH U345 | **Ordinary Differential Equations** | **Fall 2008**

Quiz 3

---

1. $\boxed{\text{10 points}}$ Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} 5 & -2 \\ 1 & 2 \end{bmatrix}$.

(a) Find the eigenvalues λ_1 and λ_2 of the matrix A.

(b) Find (non-zero) eigenvectors V_1 and V_2 corresponding to each eigenvalue.

(c) Find the general solution $Y = Y(t)$ of the given system.

(d) Find the solution $Y = Y(t)$ with initial condition $Y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

2. $\boxed{\text{10 points}}$ The matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ has eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$, with corresponding eigenvectors $V_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Sketch the phase portrait for the linear system $Y' = AY$, indicating the straight-line solutions, and at least 4 other solution curves. What kind of equilibrium point is the origin?

3. $\boxed{\text{10 points}}$ Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$.

(a) Find the eigenvalues λ_1 and λ_2 of the matrix A.

(b) Find (non-zero) eigenvectors V_1 and V_2 corresponding to each eigenvalue.

(c) Find the general solution $Y = Y(t)$ of the given system.

(d) Sketch the phase portrait for the system. What are the equilibrium points?

4. $\boxed{\text{10 points}}$ Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$.

(a) It turns out that A has a single eigenvalue, λ. Compute this eigenvalue.

(b) Given the initial condition $V_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, find the corresponding eigenvector V_1 for the eigenvalue λ.

(c) Find the solution $Y = Y(t)$ with initial condition $Y(0) = V_0$.