MTH U345 **Ordinary Differential Equations** **Fall 2008**

Practice Quiz 2

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1. Convert the following second order D.E. to a system of first order D.E.'s. **DO NOT TRY TO SOLVE** the system. All you have to do is write out the first order system.

$$y''(t) = 7y'(t) + 2y(t) + 3y^2(t).$$

2. The second order D.E. $y''(t)+9y(t) = 0$ has the general solution $y(t) = C_1 \cos(\beta t)+C_2 \sin(\beta t)$.

(a) Find β.

(b) Find C_1 and C_2 so that y satisfies the initial condition $y(0) = 4$, $y'(0) = 5$.

3. Solve the initial value problem $y'' + 7y' + 12y = 0,\ y(0) = 3\ y'(0) = -7$.

4. Solve the initial value problem $4y'' - 12y' + 9y = 0,\ y(0) = 9\ y'(0) = 8$.

5. Solve the following (partially decoupled) system:

$$\frac{dx}{dt} = 3x, \qquad \frac{dy}{dt} = -2x + 5y.$$

6. Write the following system of first order linear equations in matrix form:

$$\frac{dy_1}{dt} = 3y_1 - 7y_2, \qquad \frac{dy_2}{dt} = 5y_1 + y_2.$$

What are the equilibrium solution(s)?

7. If $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is a solution of $X' = AX$ where $A = \begin{bmatrix} 6 & -4 \\ 10 & 2 \end{bmatrix}$, give x_1' and x_2' at the point where $x_1 = 5$ and $x_2 = -3$.

8. Two of the three vector-valued functions

$$U(t) = e^{-2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad V(t) = e^{2t} \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad W(t) = e^{3t} \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

are solutions to the system $Y' = BY$ where $B = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$.

(a) Determine which two functions are solutions

(b) Using the two functions which are solutions, find a third solution Y satisfying the initial condition $Y(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

9. The system $Y' = \begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix} Y$ has solutions $Y_1(t) = e^{-5t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $Y_2(t) = e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find the solution $Y(t)$ satisfying the initial value $Y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.