MTH U345 **Ordinary Differential Equations** **Fall 2008**

Quiz 1

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1. Solve: $\dfrac{dy}{dt} = t + ty^2$, $y(0) = 1$. For which values of t is the solution defined?

$$\frac{dy}{dt} = t(1+y^2) \longrightarrow \int \frac{dy}{1+y^2} = \int t dt + C \longrightarrow \arctan y = \frac{t^2}{2} + C \longrightarrow y = \tan\left(\frac{t^2}{2} + C\right)$$

$$y(0) = 1 \longrightarrow \tan(C) = 1 \longrightarrow C = \frac{\pi}{4}$$

The solution curve is defined on an interval around $t = 0$, as long as $\dfrac{t^2}{2} + \dfrac{\pi}{4} \neq \dfrac{\pi}{2}$, that is, $t^2 \neq \dfrac{\pi}{2}$. Hence,

$$\boxed{y(t) = \tan\left(\frac{t^2}{2} + \frac{\pi}{4}\right), \quad \text{defined for } -\sqrt{\frac{\pi}{2}} < t < \sqrt{\frac{\pi}{2}}}$$

3. Solve: $\dfrac{dy}{dt} + \dfrac{3}{t}y = \dfrac{1}{t^4}$, $y(1) = 1$. For which values of t is the solution defined?

$$\mu(t) = e^{\int \frac{3}{t} dt} = e^{3\ln t} = t^3$$

$$y(t) = \frac{1}{t^3}\left(\int t^3 \cdot \frac{1}{t^4} dt + C\right) = \frac{1}{t^3}\left(\int \frac{1}{t} dt + C\right) = \frac{1}{t^3}(\ln t + C)$$

$$y(1) = 1 \longrightarrow C = 1$$

The solution curve is defined on an interval around $t = 1$, as long as $t \neq 0$. Hence, we must have $t > 0$. (This, by the way, justifies not putting absolute value forln $|t|$ in the above.) Hence,

$$\boxed{y(t) = \frac{1 + \ln t}{t^3}, \quad \text{defined for } t > 0}$$

4. A 100 gallon tank initially contains 20 gallons of pure water. A salt water solution containing 4 pounds of salt per gallon enters the tank at 7 gallons per minute, and the mixture kept uniform by stirring, flows out at the rate of 5 gallons per minute.

(a) How many gallons of salt water solution are there after t minutes?

$$V(t) = 20 + 7t - 2t = 20 + 2t$$

(b) When will the tank be full?

$$V(t) = 100 \longrightarrow t = 40$$

(c) Write down the initial value problem that describes the quantity of salt, $S(t)$ kg, at time t.

$$\frac{dS}{dt} = 28 - \frac{5}{20 + 2t}S, \quad S(0) = 0$$