MTH U345 **Ordinary Differential Equations** **Fall 2008**

Practice Quiz 1: Answer Key

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1. $\dfrac{dy}{dt} - \dfrac{2}{t}y = t^2 \sin t \quad \longrightarrow \quad y(t) = t^2(k - \cos t), \text{ for } t \neq 0$

2. $(t+1)\dfrac{dy}{dt} = 1 + y^2 \quad \longrightarrow \quad y(t) = \tan(\ln|1+t| + k), \text{ for } t \neq -1$

3. $\dfrac{dx}{dt} + \dfrac{1}{2t}x = \dfrac{1}{2} \quad \longrightarrow \quad y(t) = \dfrac{t}{3} + \dfrac{k}{\sqrt{t}}, \text{ for } t > 0$

4. $\dfrac{dy}{dt} - ty = ty^3 \quad \longrightarrow \quad y(t) = \pm\dfrac{1}{\sqrt{ke^{-t^2} - 1}}, \text{ for } t \neq \pm\sqrt{\ln k} \text{ and } k > 0$

5. $\dfrac{dx}{dt} + \dfrac{2}{t}x = \dfrac{1}{t^3} + 3, \quad x(1) = 3 \quad \longrightarrow \quad x(t) = t + \dfrac{2 + \ln t}{t^2}, \text{ for } t > 0$

6. $\dfrac{dy}{dt} + \dfrac{3}{t}y = 2, \quad y(2) = 1 \quad \longrightarrow \quad y(t) = \dfrac{t}{2}, \text{ for } t > 0$

7. $\dfrac{dy}{dt} = \dfrac{e^{2t}}{2y}, \quad y(0) = -1 \quad \longrightarrow \quad y(t) = -\sqrt{\dfrac{e^{2t}+1}{2}}, \text{ for all } t$

9. A 300 gallon tank initially contains 100 gallons of pure water. A salt water solution containing 3 pounds of salt per gallon enters the tank at 8 gallons per minute, and the mixture is removed at the rate of 6 gallons per minute. How many pounds of salt is in the tank when the tank is full?

Let $S(t)$ be the quantity of salt (in pounds) at time t. Then:

$$\frac{dS}{dt} + \frac{3}{50+t}S = 24, \; S(0) = 0 \longrightarrow S(t) = \frac{6t(100+t)\left(5000 + 100t + t^2\right)}{(50+t)^3} \longrightarrow S(100) = \frac{8000}{9}$$

10. A tank initially contains 20 kg of salt dissolved in 200 liters of water. A brine solution containing 3 kg of salt/liter flows into the tank at the rate of 2 liters/minute. The mixture, kept uniform by stirring, flows out at the rate of 4 liters/minute.

Let $S(t)$ be the quantity of salt (in kg) at time t. Then:

$$\frac{dS}{dt} + \frac{2}{100-t}S = 6, \; S(0) = 20 \longrightarrow S(t) = \frac{(100-t)(100+29t)}{500} \longrightarrow S(30) \simeq \frac{679}{5}$$

11. A microcosm contains a population of microorganisms whose birth and death rates are proportional to the square root of the population. Initially there is just 1 microorganism, but after 5 hours there are 4 of them.

Let $P(t)$ be the population at time t. Then:

$$\frac{dP}{dt} = k\sqrt{P} \cdot P, \; P(0) = 1 \longrightarrow P(t) = \frac{4}{(kt-2)^2}$$

$P(5) = 4 \longrightarrow k = \dfrac{1}{5}$ or $k = \dfrac{3}{5}$, but only the former makes sense (why?)

thus, $P(t) = \dfrac{100}{(t-10)^2}$

since $\lim\limits_{t \to 10^-} P(t) = \infty$, the microorganisms take over the microcosm at time $t = 10$