Prof. Alexandru Suciu

MATH 7221 **TOPOLOGY 2** **Fall 2018**

Homework 1

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1. A *flag complex* is an (abstract) simplicial complex K all of whose minimal non-faces are two-element sets. Show that K is a flag complex if and only if the following condition holds: if all of the edges of a potential face are in K, then that face must also be in K.

2. Let G be a (finite, simple) graph. A *clique* in G is a complete subgraph of G, i.e., a subgraph where any two vertices are connected by an edge. The *clique complex* of G, denoted $\Delta(G)$, is the simplicial complex whose vertex set is the vertex set of G, and whose faces are the cliques of G.

(a) Show that $\Delta(G)$ is indeed an (abstract) simplicial complex, and, in fact, that $\Delta(G)$ is a flag complex.

(b) Conversely, show that any flag complex K is the clique complex of its 1-skeleton (i.e., the graph $G = K^{(1)}$ consisting of the vertices and edges of K).

3. Let K be a finite simplicial complex, and let $|K|$ be its geometric realization.

(a) Show that $|K|$ Hausdorff.

(b) Show that $|K|$ is compact.

4. Let T^2 be the 2-torus.

(a) Show that the minimum number of vertices in a triangulation of T^2 is 7.

(b) Construct a simplicial complex K on 7 vertices such that $|K|$ is homeomorphic to T^2.

5. From Hatcher's book: Chapter 1.A, Page 86, Problem 3.

6. From Hatcher's book: Chapter 2.1, Page 133, Problem 23.

Bonus question: Can you find any connection between Problems 2 and 6?