Prof. Alexandru Suciu

MATH 4565 | **TOPOLOGY** | **Spring 2018**

FINAL EXAM

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1. Let X and Y be topological spaces, and $\mathrm{pr}_1 \colon X \times Y \to X$ the first-coordinate projection map. Prove or disprove the following assertions.

(a) pr_1 is a continuous map.

(b) pr_1 is an open map.

(c) pr_1 is a closed map.

(d) pr_1 is a quotient map.

2. Let $X = \{1, 2, 3, 4\}$, endowed with the topology

$$\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}.$$

Let $Y = \{a, b, c\}$, and $f\colon X \to Y$ the function sending $1 \mapsto a$, $2 \mapsto b$, $3 \mapsto b$, $4 \mapsto c$. Find the quotient topology on Y defined by the map f.

3. Let X be a topological space. Suppose X contains an infinite, closed, discrete subspace. Show that X is *not* compact.

4. Let $f\colon X \to Y$ be a quotient map. Suppose Y is connected, and, for each $y \in Y$, the subspace $f^{-1}(\{y\})$ is connected. Show that X is also connected.

5. A subspace $A \subset X$ is called a *deformation retract* of X if there is a retraction $r\colon X \to A$ with the property that $i \circ r \simeq \mathrm{id}_X$. Prove the following: if A is a retract of a contractible space X, then A is a deformation retraction of X.

6. Let f and g be two paths in $\mathbb{R}^2 \setminus \{(0,0)\}$.

(a) Show that f is homotopic to g.

(b) Suppose f and g both start at $(-1,0)$ and end at $(0,1)$. Is f always path-homotopic to g?

7. Let $p\colon E \to B$ be a covering map. Suppose E is path-connected, and $\pi_1(B, b_0) = 0$. Show that p is a homeomorphism.

8. Let $X = S^1 \bigvee S^1$ be the union of two circles at the basepoint x_0, let a and b be loops at x_0 going around once each of those circles. Draw 4 non-equivalent, connected 3-fold covers of X. In each case:

(a) Indicate the lifts of the base curves.

(b) Determine the permutations associated to the loops a and b by the lifting correspondence.

(c) Indicate whether the cover is regular or not.